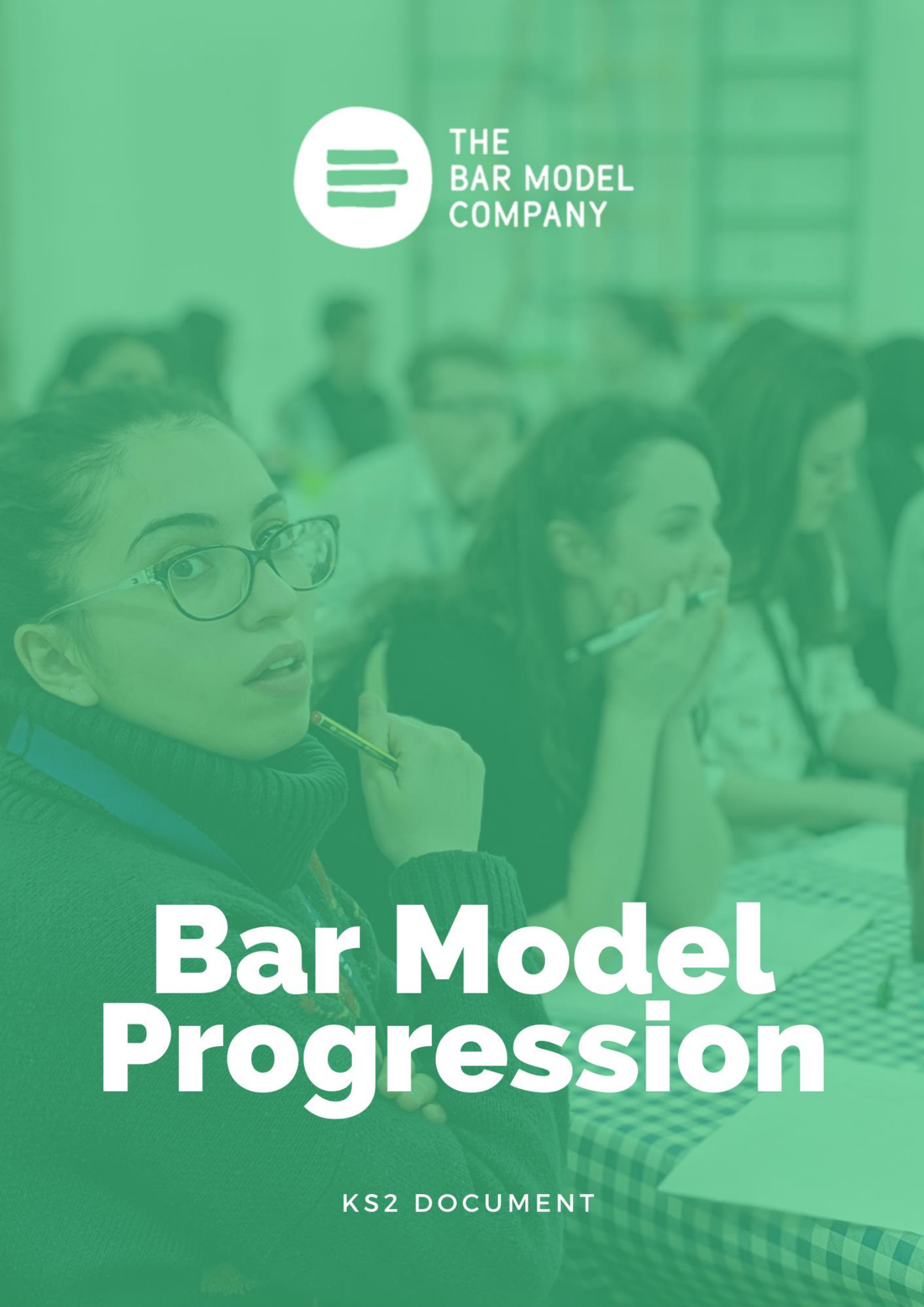


THE
BAR MODEL
COMPANY

Bar Model Progression

KS2 DOCUMENT

Introduction

This bar model progression document is written for teachers who are implementing or plan to implement, bar modelling as a visual tool in KS2. With detailed examples, this document provides insights into implementing bar modelling in the three stages of learning using the Concrete-Pictorial-Abstract approach. The mathematics framework used in this document is in accordance with the mathematics programmes of study found in the National Curriculum in England.

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Year 3

BAR MODELLING PROGRESSION

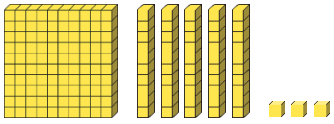
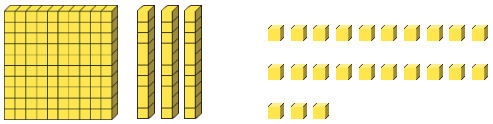
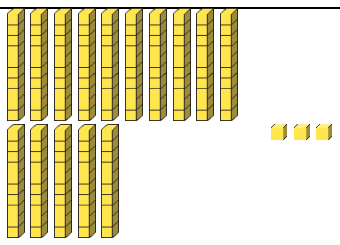
Bar modelling in Year 3 mathematics framework

1.1. Year 3 Whole Number up to 1000

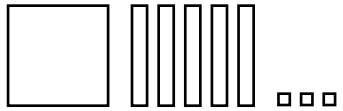
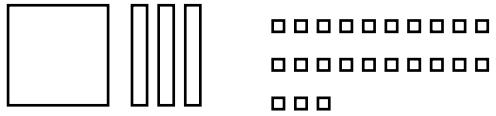
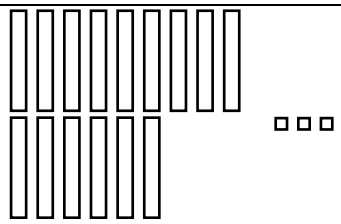
1.1.1. Year 3 Number sense by counting to 1 000

Learning experience:

- count from 0 in multiples of 4, 8, 50 and 100; find 10 or 100 more or less than a given number
- recognise the place value of each digit in a three-digit number (hundreds, tens, ones)
- compare and order numbers up to 1000
- identify, represent and estimate numbers using different representations
- read and write numbers up to 1000 in numerals and in words
- solve number problems and practical problems involving these ideas.

Learning Objective **for counting numbers**	**Example**
Students are able to: • use Cuisenaire rod and base ten blocks to demonstrate a quantity, cardinal numbers • compare and order numbers, using '<', '>' and '=' signs • Count forward / backward in 4s, 8s, 50s and 100s by using base ten unit cubes, rods and hundred blocks or Cuisenaire rods • partitioning related to place value • draw part-part-whole bar model to represent Cuisenaire rods and base ten blocks	**<u>Hands-on concrete stage: Using Cuisenaire rods and base ten units/rods/blocks to carry out partitioning of a number related to place value</u>** Example: Students use different ways to partition 153 1 hundred, 5 tens, 3 ones 100 + 50 + 3  1 hundred, 3 tens, 23 ones 100 + 30 + 23  15 tens, 3 ones 150 + 3 

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Learning Objective for counting numbers	Example
	<u>Pictorial stage: Using rectangles and squares to show different ways of number partitioning</u> Example: Students identify numbers from diagrams 1 hundred, 5 tens, 3 ones 100 + 50 + 3  1 hundred, 3 tens, 23 ones 100 + 30 + 23  15 tens, 3 ones 150 + 3  **<u>Abstract stage: Using part-part-whole bar model to show different ways of number partitioning</u>** Example: Students use different ways to partition 153 153 100 | 50 | 3 153 100 | 30 | 23 153 150 | 3

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Year 3 Addition / Subtraction of numbers up to 1 000

Learning experience:

- add and subtract numbers mentally, including:
  - a three-digit number and ones
  - a three-digit number and tens
  - a three-digit number and hundreds
- add and subtract numbers with up to three digits, using formal written methods of columnar addition and subtraction
- estimate the answer to a calculation and use inverse operations to check answers
- solve problems, including missing number problems, using number facts, place value, and more complex addition and subtraction.

<table>
<tr><th>Learning objective for addition</th><th>Example</th></tr>
<tr><td rowspan="2">Students are able to:

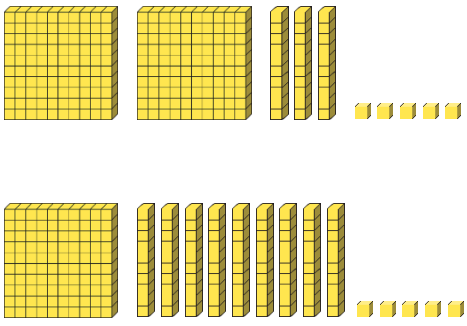
• add three-digit numbers with ones/tens/hundreds without regrouping
• add three-digit numbers with ones/tens/hundreds with regrouping
• solve problems using part-part-whole bar model</td><td><u>Hands-on concrete stage: Using 3D cubes and rods to represent a number in word problem</u>

Example: Students model the word problem question using base ten blocks.

235 people visited the library on Saturday.

198 people visited the library on Sunday.

How many people visited the library on both days?



Students regroup 13 ones to 1 ten 3 ones and 12 tens to 1 hundred and 2 tens</td></tr>
<tr><td><u>Pictorial stage: Using rectangles and squares to represent base ten blocks</u></td></tr>
</table>

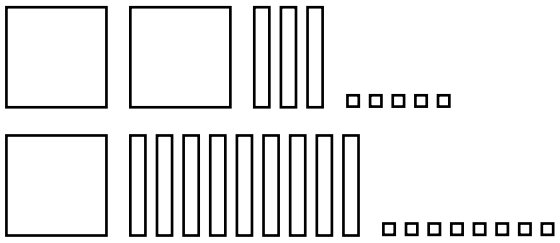
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Example: Students model the word problem question with diagrams of base ten blocks.

235 people visited the library on Saturday.

198 people visited the library on Sunday.

How many people visited the library on both days?



Students regroup 13 ones to 1 ten 3 ones and 12 tens to 1 hundred and 2 tens

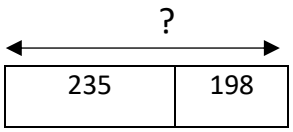
<u>Abstract stage: Using part-part-whole bar model to solve word problem</u>

Example: Students model the word problem question with part-part-whole bar model.

235 people visited the library on Saturday.

198 people visited the library on Sunday.

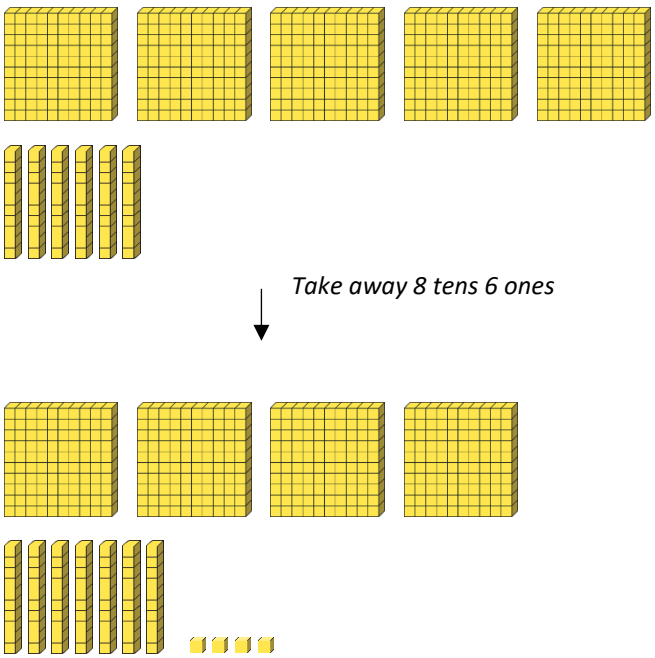
How many people visited the library on both days?



235 + 198 = 433

	1	*1*	
	2	3	5
+	1	9	8
	4	3	3

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Learning Objective for subtraction (Taking away)	Example
Students are able to: • subtract three-digit numbers with ones/tens/hundreds without regrouping • subtract three-digit numbers with ones/tens/hundreds with regrouping • solve word problems (taking away) using part-part-whole bar model	**<u>Hands-on concrete stage: Using 3D cubes and rods to represent a number in word problem</u>** Example: Students model the word problem question using base ten blocks. *There were 560 passengers on a train.* *When the train reached a station, 86 passengers alighted.* *How many passengers were left on the train?*  *Students regroup 1 ten to 10 ones to take away 6 and regroup 1 hundred to 10 tens to take away 80.*
	<u>Pictorial stage: Using rectangles and squares to represent base ten blocks</u>

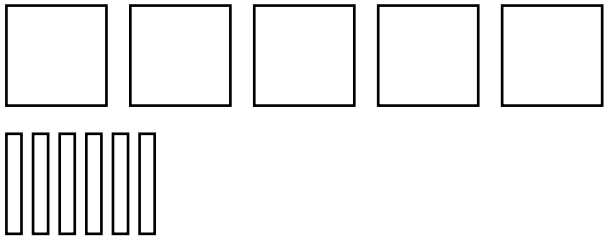
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Example: Students model the word problem question with diagrams of base ten blocks and cross out ones that are removed.

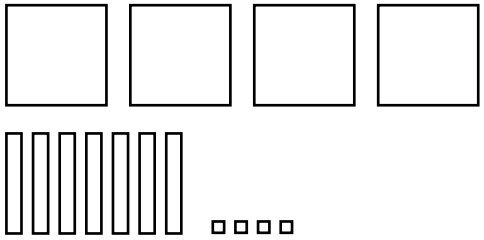
There were 560 passengers on a train.

When the train reached a station, 86 passengers alighted.

How many passengers were left on the train?



Take away 8 tens 6 ones



Students regroup 1 ten to 10 ones to take away 6 and regroup 1 hundred to 10 tens to take away 80.

Abstract stage: Using part-part-whole bar model to solve word problem

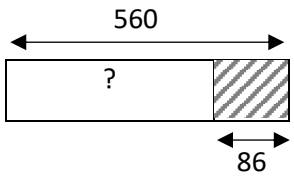
Example: Students model the word problem question with part-part-whole bar model, shading the part to be removed.

There were 560 passengers on a train.

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When the train reached a station, 86 passengers alighted.

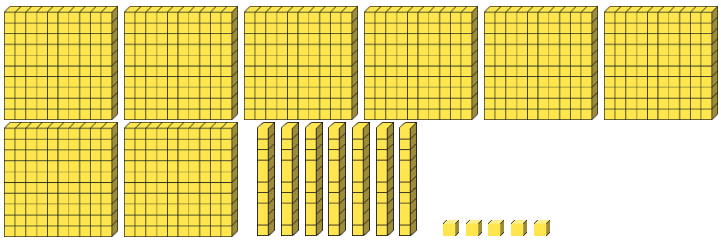
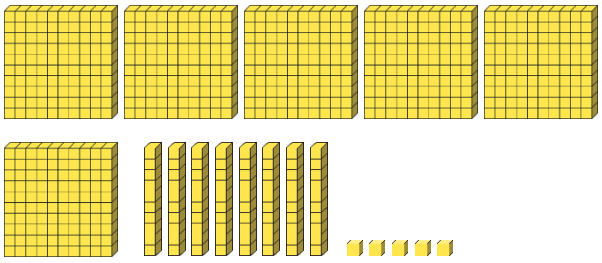
How many passengers were left on the train?



	4	*15*	
	~~5~~	~~6~~	$^{1}0$
-		8	6
	4	7	4

560 – 86 = 474

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Learning Objective for subtraction (difference)	Example
Students are able to: • subtract three-digit numbers with ones/tens/hundreds without regrouping • subtract three-digit numbers with ones/tens/hundreds with regrouping • solve word problems (taking away) using comparison bar model	**<u>Hands-on concrete stage: Using 3D cubes and rods to represent a number in word problem</u>** Example: Students model the word problem question using base ten blocks and Cuisenaire rods for comparison. *A sofa set cost £685.* *A dining table set cost £875.* *How much more does the dining table cost than the Sofa?*   *Students regroup 1 hundred to 10 tens to take away 80.*

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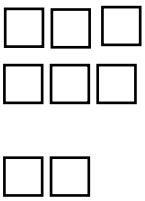
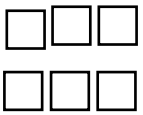
Pictorial stage: Using rectangles and squares to model comparison between two numbers to find difference

Example: Students model the word problem question using base ten blocks and Cuisenaire rods for comparison.

A sofa set cost £685.

A dining table set cost £875.

How much more does the dining table cost than the Sofa?

	hundreds	tens	ones
Dining table £875			
Sofa set £685			

Students regroup 1 hundred to 10 tens to take away 80.

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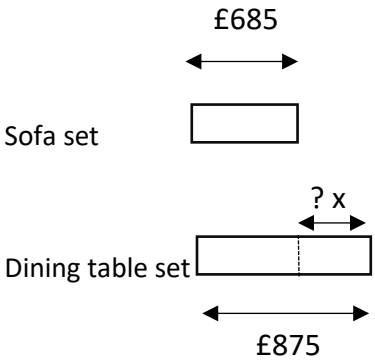
Abstract stage: Using rectangles and squares to model comparison between two numbers to find difference

Example: Students model the word problem question using base ten blocks and Cuisenaire rods for comparison.

A sofa set cost £685.

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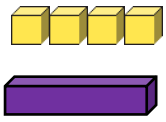
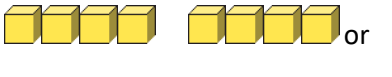
$$\begin{array}{rrrr} & {}^{7} & & \\ & \cancel{8} & {}^{1}7 & 5 \\ - & 6 & 8 & 5 \\ \hline & 1 & 9 & 0 \\ \hline \end{array}$$

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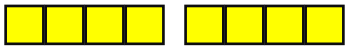
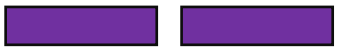
1.1.2. Year 3 Simple Multiplication / Division of numbers 0 – 1 000

Learning experience:

- recall and use multiplication and division facts for the 3, 4 and 8 multiplication tables
- write and calculate mathematical statements for multiplication and division using the multiplication tables that they know, including for two-digit numbers times one-digit numbers, using mental and progressing to formal written methods
- solve problems, including missing number problems, involving multiplication and division, including positive integer scaling problems and correspondence problems in which objects are connected to m objects

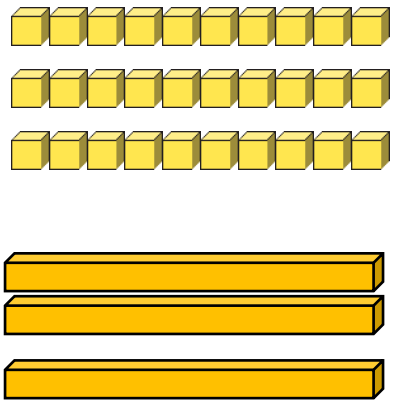
Learning Objective **for multiplication (*x times*)**	**Example**
Students are able to: • relate positive integer scaling problems to multiplication (twice, double, thrice, 4 times, 8 times) • solve word problems using comparison bar model	**<u>Hands-on concrete stage: Demonstrate scaling problems to multiplication using base ten cubes / Cuisenaire rods</u>** Example: Students use base ten cubes / Cuisenaire rod to explain the concept of twice/double, thrice, x times *Miss Mary has 4 muffins.* *Her sister has twice as many muffins as Mary.* *How many muffins does Mary's sister have?* *Mary*  or *Sister*  or  or 

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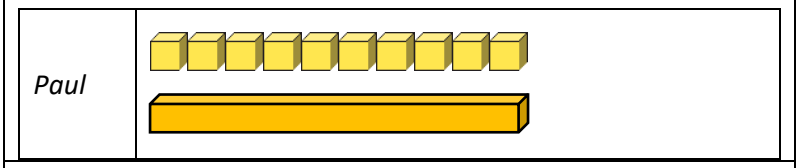
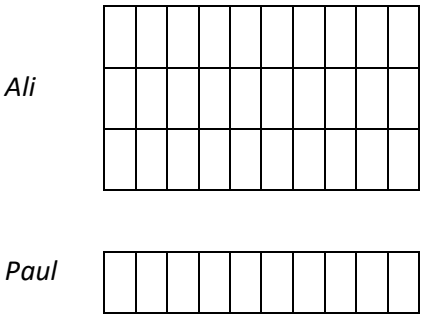
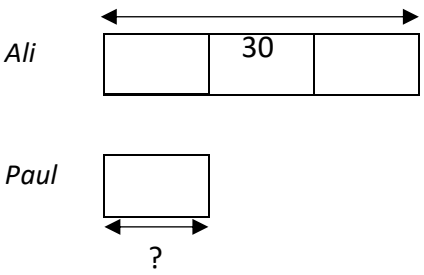
Learning Objective for multiplication (*x times*)	Example
	<u>Pictorial stage: Demonstrate scaling problems to multiplication using rectangles and squares</u> Example: Students draw rectangles/squares to explain the concept of twice/double, thrice, x times *Miss Mary has 4 muffins.* *Her sister has twice as many muffins as Mary.* *How many muffins does Mary's sister have?* *Mary*  *Sister*  ***Or*** *Mary*  *Sister* 
	<u>Abstract stage: Demonstrate scaling problems to multiplication using comparison bar model</u> Example: Students draw comparison bar model to explain the concept of twice/double, thrice, x times *Miss Mary has 4 muffins.* *Her sister has twice as many muffins as Mary.* *How many muffins does Mary's sister have?*

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Learning Objective for multiplication (*x times*)	Example
	Mary 4 *Sister* 4 4 ? *2 x 4 = _____*

Learning Objective for division	Example
Students are able to: • relate positive integer scaling problems to division (twice, double, thrice, 4 times, 8 times) using inverse operation of x / ÷ • solve word problems using comparison bar model	**<u>Hands-on concrete stage: Demonstrate scaling problems to division using base ten cubes / Cuisenaire rods</u>** Example: Students use base ten cubes / Cuisenaire rod to explain the concept of inverse operation (x , ÷) *Ali has 30 conkers.* *Ali has thrice as many conkers as Paul.* *How many conkers does Paul have?* *Ali* 

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Learning Objective for division	Example
	
	<u>Pictorial stage: Demonstrate scaling problems to division using rectangles and squares</u> Example: Students draw rectangles/squares to explain the concept of inverse operation (x , ÷) 
	<u>Abstract stage: Demonstrate scaling problems to division using comparison bar model</u> Example: Students draw comparison bar model to explain the concept of inverse operation (x , ÷) *Ali has 30 conkers.* *Ali has thrice as many conkers as Paul.* *How many conkers does Paul have?*  *_____ x 3 = 30* *30 ÷ 3 = _____*

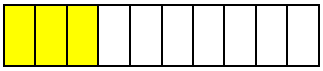
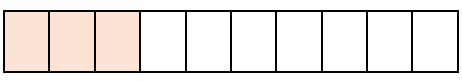
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1.2. Year 3 Fraction

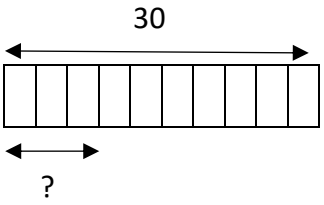
Learning experience:

- count up and down in tenths; recognise that tenths arise from dividing an object into 10 equal parts and in dividing two-digits numbers or quantities by 10
- recognise, find and write fractions of a discrete set of objects: unit fractions and non-unit fractions with small denominators
- recognise and use fractions as numbers: unit fractions and non-unit fractions with small denominators
- recognise and show, using diagrams, equivalent fractions with small denominators
- add and subtract fractions with the same denominator within one whole [for example, $\frac{5}{7} + \frac{1}{7} = \frac{6}{7}$]
- compare and order unit fractions, and fractions with the same denominators
- solve problems that involve all of the above

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Learning Objective For fraction	Examples
Students are able to: • count up and down in tenths; recognise that tenths arise from dividing an object into 10 equal parts and in dividing two-digits numbers or quantities by 10 • add and subtract tenths within one whole • draw part-part-whole bar model to demonstrate multiple of tenth of a 2-digits number	**<u>Hands-on concrete stage: Cut strips of paper of different lengths (multiple of 10s) into 10 equal smaller groups (denominator) then select number of required pieces (numerator)</u>** Example: Students use strips of paper (different length) to cut into 10 equal parts. 10 cm Paper strip  20 cm Paper strip  30 cm Paper strip  After cutting the pieces, students count each piece as 1 tenth. Students observed that a tenth of a longer strip of paper is longer than a tenth of a shorter strip of paper. *3 tenths = 3 pieces of each paper strip of same colour*
	<u>Pictorial stage: Draw rectangles of different lengths (multiple of 10), divide into 10 smaller parts (denominator) then shade number of required parts (numerator)</u> Example: Students draw rectangles of different length (multiple of 10), divide into 10 equal part and shade the required parts.  $\frac{3}{10}$  

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	<u>Abstract stage: Draw part-part-whole bar model to represent a fraction of a number</u> Example: Students draw bar model of tenths of different number in multiple of tens. *10 tenths = 30* *1 tenth = 30 ÷ 10* *= 3* *3 tenths = 3 x 3* *? = 9*  ?

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1.3. Year 3 Measurement (Length/Mass/Volume/Capacity/Money)

National Curriculum content:

Learning experience:

- measure, compare, add and subtract: lengths (m/cm/mm); mass (kg/g); volume/capacity (l/ml)
- measure the perimeter of simple 2-D shapes
- add and subtract amounts of money to give change, using both £ and p in practical contexts
- tell and write the time from an analogue clock, including using Roman numerals from I to XII, and 12-hour and 24-hour clocks
- estimate and read time with increasing accuracy to the nearest minute; record and compare time in terms of seconds, minutes and hours; use vocabulary such as o'clock, am/pm, morning, afternoon, noon and midnight
- know the number of seconds in a minute and the number of days in each month, year and leap year
- compare durations of events [for example, to calculate the time taken by particular events or tasks]

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Learning Objective For length	Examples
Students are able to: • Compare measures (length/mass/volume/capacity/money) using simple scaling by integers (for example, a given quantity or measure is twice as long or 5 times as high) and this connects to multiplication • translate measurement (length/mass/volume/capacity/money) of objects onto comparison bar model	**<u>Hands-on concrete stage: Count money in multiple of 10-pence (relate to fraction topic on tenth)</u>** Example: Students count amount of money using ten-pence (unit). £1 = ten 10-pence £1 = A hundred 1-pence $\frac{1}{10}$ of £1 = 10-pence $\frac{1}{10}$ of 100 pence = 10-pence Enter answer in the empty boxes.

	Twice 2x	Thrice 3x	Half $\frac{1}{2}$	Tenth $\frac{1}{10}$
10p 10p 10p 10p 10p 10p				
10p 10p 10p 10p 10p 10p 10p 10p 10p 10p				
10p 10p 10p 10p 10p 10p 10p 10p 10p 10p 10p 10p				
10p 10p 10p 10p 10p 10p 10p 10p 10p 10p 10p 10p 10p 10p 10p 10p 10p 10p 10p 10p				

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Pictorial stage: Draw coins as fraction of an amount in multiple of 1o-pence (relate to fraction topic on tenth)

Example: Students draw number of 10-pence coins to show amount of money.

$\frac{1}{2}$ of 80-pence	(10p)(10p)(10p)(10p)
$\frac{1}{10}$ of £3	
Thrice of £1.20 = Thrice of twelve 10-pence	

Abstract stage: Compare using comparison bar model

Example: Students draw comparison bar model from word problem solving.

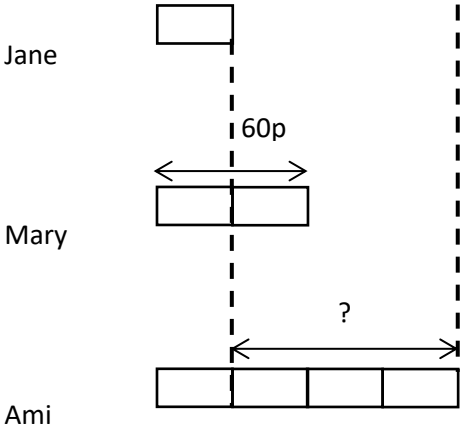
Mary has 60p.

Ami has twice as much money as Mary.

Jane has $\frac{1}{2}$ the amount of what Mary has.

How much more does Ami have more than Jane?

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2 units = 60p
1 unit = 60p ÷ 2
= 30p
3 units = 3 x 30p
= 90p

Students see each unit as a box for equal number of coins.

60p as six 10-pence coins.

Jane has half of six 10-pence coins -> three 10-pence coins

Ami has twice as much as six 10-pence coins -> twelve 10-pence coins

Ami has 12 – 3 = 9 more 10-pence coins than Jane.

OR

Ami had 3 groups of three 10-pence more than Jane -> 90p

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Year 4

BAR MODELLING PROGRESSION

Bar modelling in Year 4 mathematics framework

1.1. Year 4 Whole Number up to 10 000

1.1.1. Year 4 Number sense by counting to 10 000

Learning experience:

- count in multiples of 6, 7, 9, 25 and 1,000
- find 1,000 more or less than a given number
- count backwards through 0 to include negative numbers
- recognise the place value of each digit in a four-digit number (1,000s, 100s, 10s, and 1s)
- order and compare numbers beyond 1,000
- identify, represent and estimate numbers using different representations
- round any number to the nearest 10, 100 or 1,000
- solve number and practical problems that involve all of the above and with increasingly large positive numbers
- read Roman numerals to 100 (I to C) and know that over time, the numeral system changed to include the concept of 0 and place value

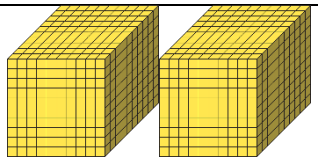
<table>
<tr><th>Learning Objective

for counting numbers</th><th>Example</th></tr>
<tr><td>Students are able to:

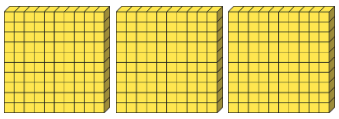
• use Cuisenaire rod and base ten blocks to demonstrate a quantity, cardinal numbers
• compare and order numbers, using ‘<’, ‘>’ and ‘=’ signs
• Count forward / backward in 6s, 7s, 9s, 25s and 1,000s by using base-ten unit cubes, rods and hundred blocks or Cuisenaire rods
• partitioning related to place value
• draw part-part-whole bar model to represent Cuisenaire rods and base ten blocks</td><td><u>Hands-on concrete stage: Using Cuisenaire rods and base ten units/rods/blocks/cubes to carry out partitioning of a number related to place value</u>

Example: Students use different ways to partition 2351

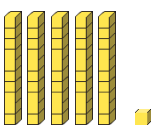
Without regrouping:



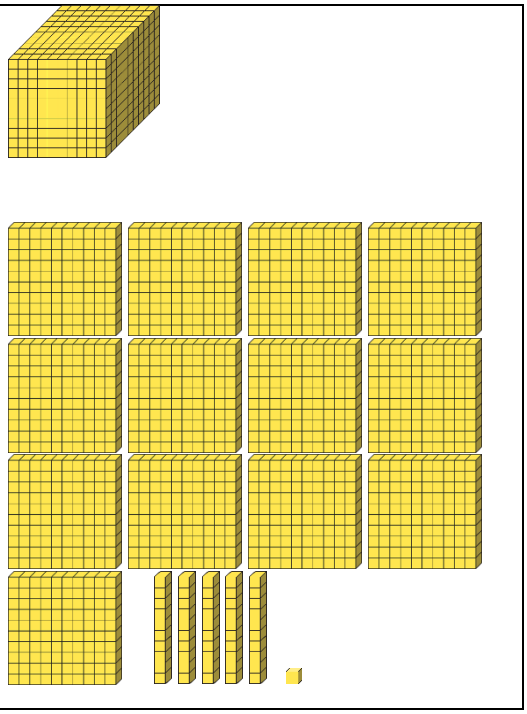
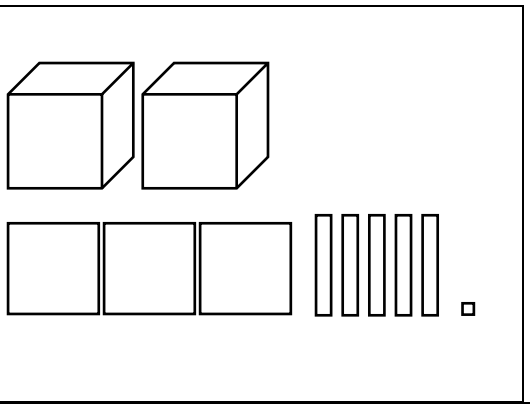
2 thousands, 3 hundreds, 5 tens, 3 ones



2000 + 300 + 50 + 1

</td></tr>
</table>

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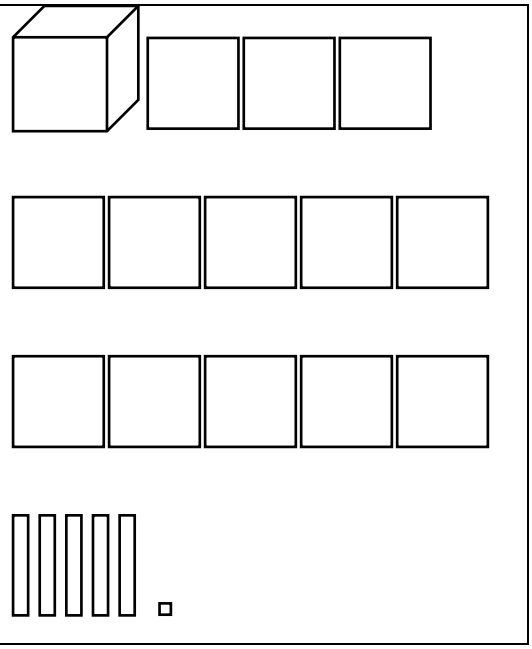
Learning Objective for counting numbers	Example
	With regrouping: 1 thousand, 13 hundreds, 5 tens, 1 one 1000 + 1300 + 50 + 1 
	<u>Pictorial stage: Using rectangles and squares to show different ways of number partitioning</u> Example: Students identify numbers from diagrams Without regrouping: 2 thousands, 3 hundreds, 5 tens, 3 ones 2000 + 300 + 50 + 1 

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<table>
<tr><th>Learning Objective

for counting numbers</th><th>Example</th></tr>
<tr><td></td><td>
<table>
<tr><td>With regrouping:

1 thousand, 13 hundreds, 5 tens, 1 one

1000 + 1300 + 50 + 1</td><td></td></tr>
</table>

<u>Abstract stage: Using part-part-whole bar model to show different ways of number partitioning</u>

Example: Students use different ways to partition 2351

<table>
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<tr><td>2000</td><td>300</td><td>50</td><td>1</td></tr>
</table>

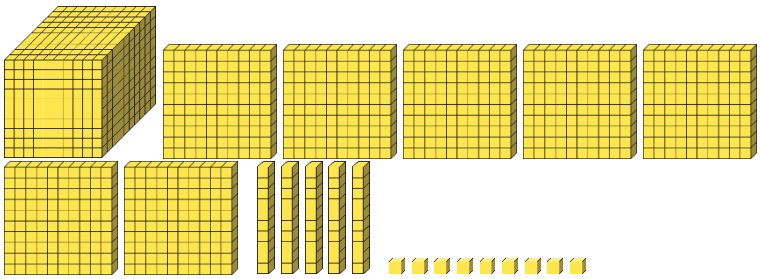
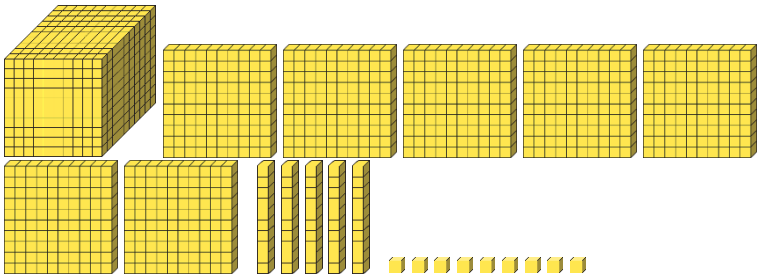
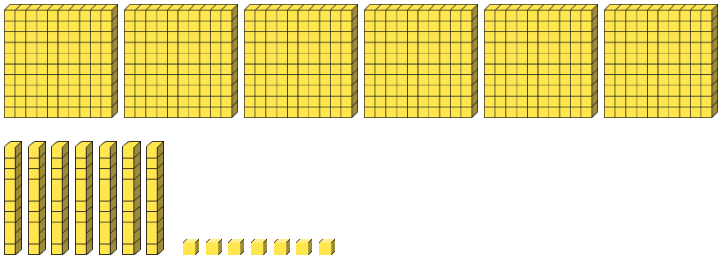
<table>
<tr><td colspan="4">2351</td></tr>
<tr><td>1000</td><td>1300</td><td>50</td><td>1</td></tr>
</table>
</td></tr>
</table>

1.1.2. Year 4 Addition / Subtraction of numbers up to 10 000

Learning experience:

- add and subtract numbers with up to 4 digits using the formal written methods of columnar addition and subtraction where appropriate
- estimate and use inverse operations to check answers to a calculation
- solve addition and subtraction two-step problems in contexts, deciding which operations and methods to use and why

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Learning Objective for addition	Example
Students are able to: • add four-digit numbers with ones/tens/hundreds/thousands without regrouping • add four-digit numbers with ones/tens/hundreds/thousands with regrouping • solve problems using comparison bar model	**<u>Hands-on concrete stage: Using 3D cubes to represent a number in word problem</u>** Example: Students model the word problem question using base ten blocks. *1759 oranges were harvested last year.* *677 more apples were harvested in the same year.* *How many fruits were harvested in total?* Number of oranges:  Number of apples:  and 

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Students notice that to find number of apples, they have to form the same number as oranges, then add 677 more base-ten blocks. Then, they add 1759 apples to find total number of fruits harvested.

Pictorial stage: Using number disks to represent base ten blocks

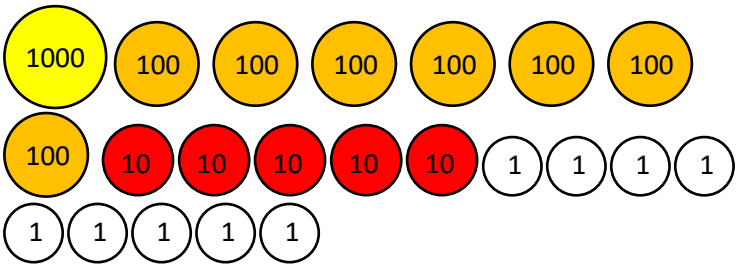
Example: Students model the word problem question with number disks.

1759 oranges were harvested last year.

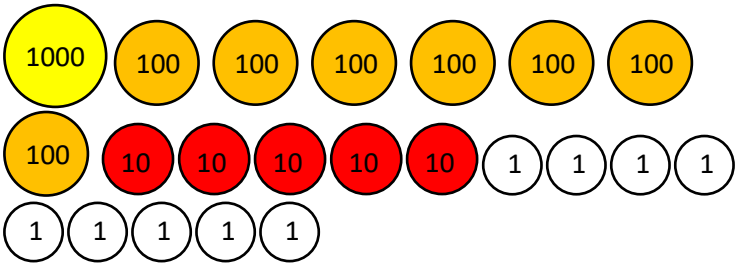
677 more apples were harvested in the same year.

How many fruits were harvested in total?

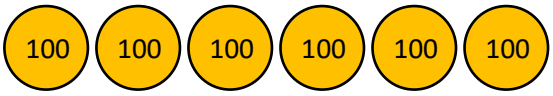
No. of oranges:



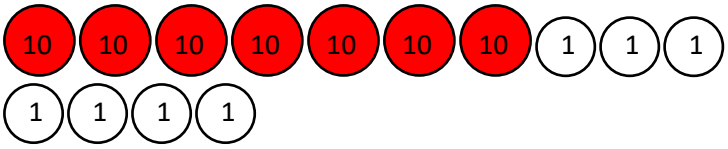
No. of apples:



and



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Students notice that to find number of apples, they have to form the same number as oranges, then add 677 more base-ten blocks. To complete the answer to the question they would then add 1759 apples to find total number of fruits harvested.

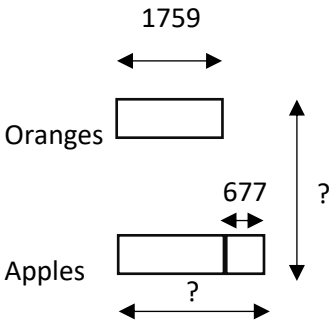
Abstract stage: Using comparison bar model to solve word problem

Example: Students model the word problem question with comparison bar model.

1759 oranges were harvested last year.

677 more apples were harvested in the same year.

How many fruits were harvested in total?



	1	*1*	*1*	
	1	7	5	9
+		6	7	7
	2	4	3	6

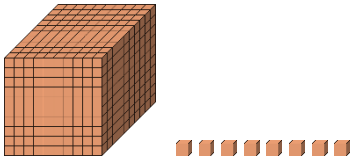
	1		*1*	
	2	4	3	6
+	1	7	5	9
	4	1	9	5

Students draw the same length of rectangles on both oranges and apples. Then add a shorter rectangle of 677 to apples to show there are 677 more apples.

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Learning Objective for subtraction (difference)	Example
Students are able to: • subtract four-digit numbers with ones/tens/hundreds/thousands without regrouping • subtract four-digit numbers with ones/tens/hundreds/thousands with regrouping • solve problems using comparison bar model	**<u>Hands-on concrete stage: Using 3D cubes to represent a number in word problem</u>** Example: Students model the word problem question using base ten blocks for comparison. *2036 apples and 1709 oranges were harvested last year.* *456 apples and 1008 oranges were harvested this year.* *Which year has the most fruits harvested? How much more?* No. of apples last year: No. of oranges last year: No. of apples this year: No. of oranges this year:

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Students add the number of apples and oranges in each year. Compare the numbers and subtract.

Fruits last year: 2036 + 1709 = 3745

Fruits this year: 456 + 1008 = 1464

No. of more fruits harvested last year

= 3745 – 1464

= 2281

Pictorial stage: Using number disks to represent base ten blocks

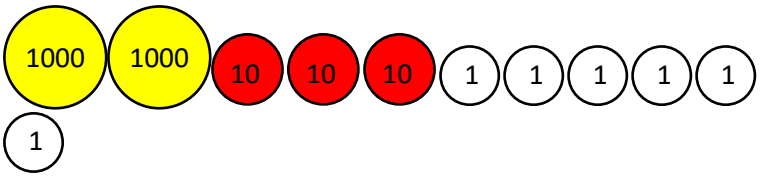
Example: Students model the word problem question with number disks.

2036 apples and 1709 oranges were harvested last year.

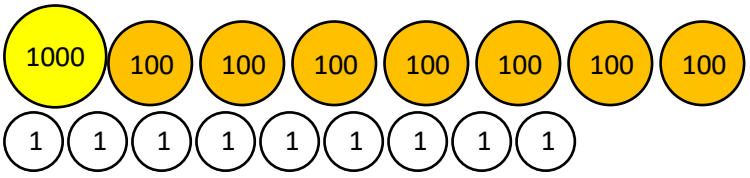
456 apples and 1008 oranges were harvested this year.

Which year has the most fruits harvested? How much more?

No. of apples last year:

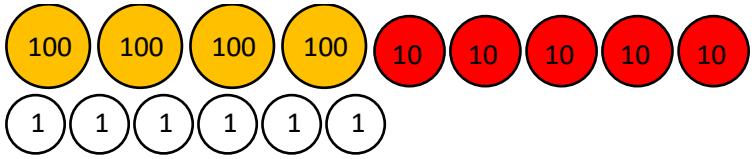


No. of oranges last year:

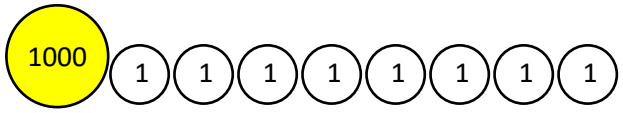


No. of apples this year:

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No. of oranges this year:



Students add the number of apples and oranges in each year. Compare the numbers and subtract.

Fruits last year: 2036 + 1709 = 3745

Fruits this year: 456 + 1008 = 1464

No. of more fruits harvested last year

= 3745 – 1464

= 2281

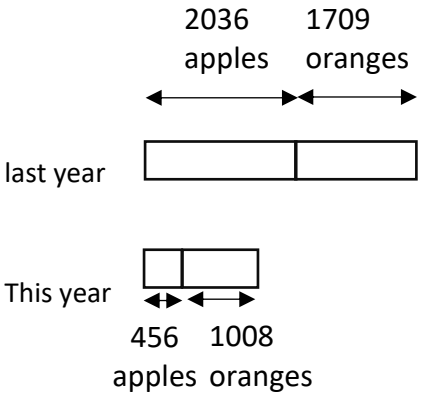
Abstract stage: Using comparison bar model to solve word problem

Example: Students model the word problem question using comparison bar model.

2036 apples and 1709 oranges were harvested last year.

456 apples and 1008 oranges were harvested this year.

Which year has the most fruits harvested? How much more?



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No. of fruits last year:

			1	
	2	0	3	6
+	1	7	0	9
	3	7	4	5

No. of fruits this year:

			1	
		4	5	6
+	1	0	0	8
	1	4	6	4

Difference:

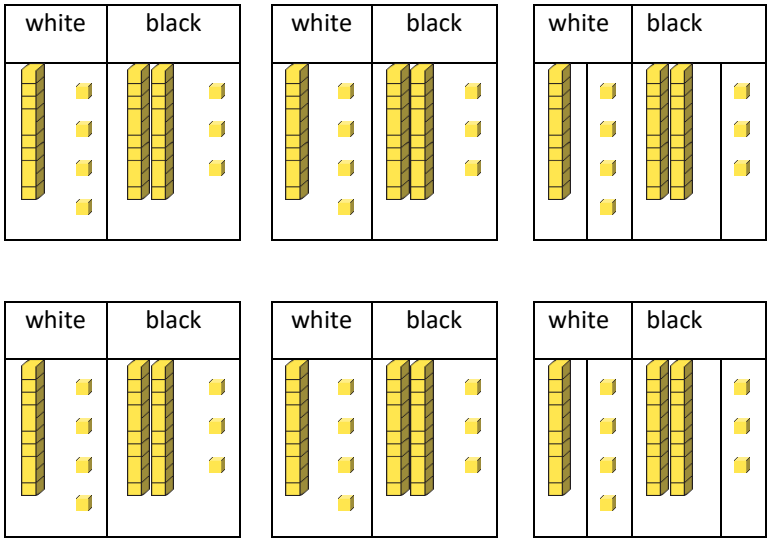
		6		
	3	~~7~~	*1*4	5
-	1	4	6	4
	2	2	8	1

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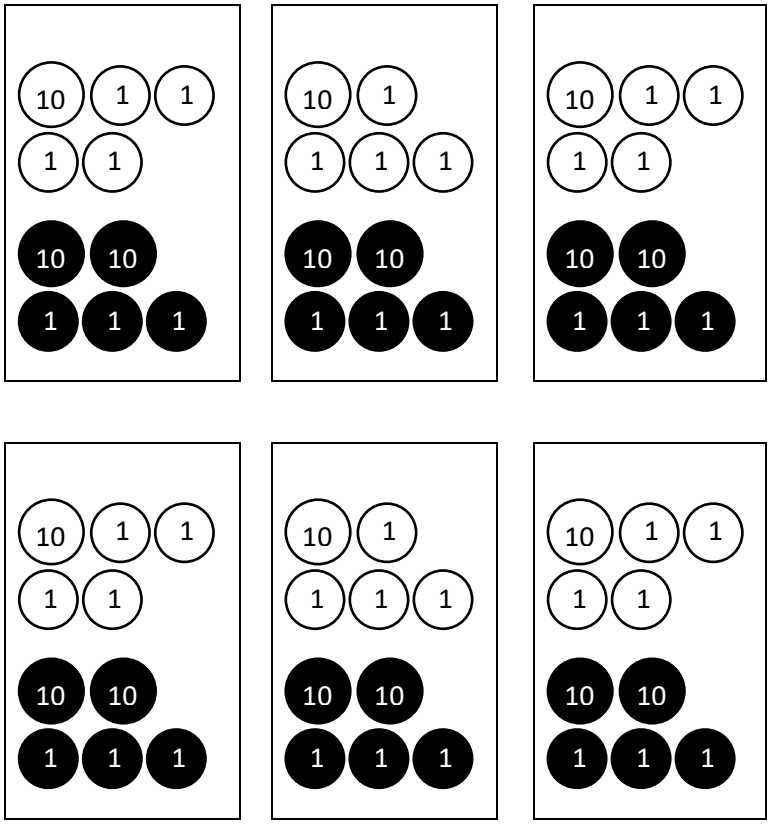
1.1.3. Year 4 Multiplication / Division of numbers 0 – 10 000

Learning experience:

- recall multiplication and division facts for multiplication tables up to 12 × 12
- use place value, known and derived facts to multiply and divide mentally, including: multiplying by 0 and 1; dividing by 1; multiplying together 3 numbers
- recognise and use factor pairs and commutativity in mental calculations
- multiply two-digit and three-digit numbers by a one-digit number using formal written layout
- solve problems involving multiplying and adding, including using the distributive law to multiply two-digit numbers by 1 digit, integer scaling problems and harder correspondence problems such as n objects are connected to m objects

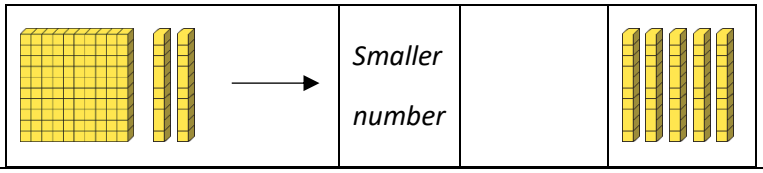
Learning Objective **for multiplication (groups of)**	**Example**
Students are able to: • solve problems involving multiplying and adding, including using the distributive law to multiply two-digit numbers by 1 digit, using part-part-whole bar model	**<u>Hands-on concrete stage: Demonstrate multiplication with 'groups of' using base ten cubes / Cuisenaire rods</u>** Example: Students use base ten cubes / Cuisenaire rod to explain the 'groups of' concept. *There are 14 white buttons and 23 black buttons in each box. How many buttons are there in 6 such boxes?*  *6 groups of 37 buttons = 6 x 37 = 37 x 6*

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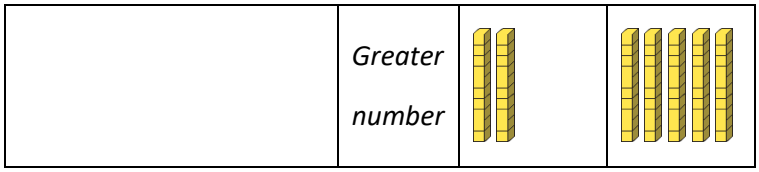
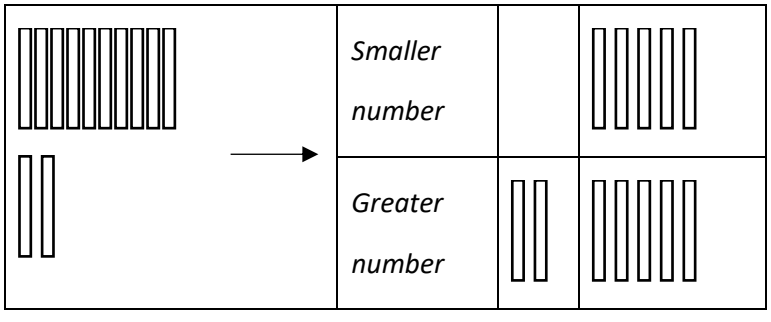
Learning Objective for multiplication (groups of)	Example
	<u>Pictorial stage: Demonstrate multiplication with 'groups of' using number disks</u> Example: Students draw number disks to explain the 'groups of' concept. *There are 14 white buttons and 23 black buttons in each box. How many buttons are there in 6 such boxes?*  *6 groups of 37 buttons = 6 x 37 = 37 x 6*
	<u>Abstract stage: Demonstrate multiplication with 'groups of using part-part-whole bar model</u> Example: Students draw part-part-whole bar model to explain the 'groups of' concept.

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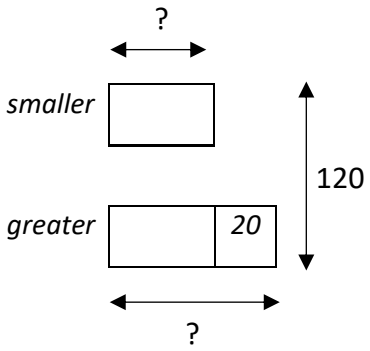
Learning Objective for multiplication (groups of)	Example
	There are 14 white buttons and 23 black buttons in each box. How many buttons are there in 6 such boxes? ? 14+23 | 14+23 | 14+23 | 14+23 | 14+23 | 14+23 No. of buttons in 1 box: 1 4 + 2 3 3 7 No. of buttons in 6 boxes: *4* 3 7 x 6 2 2 2

Learning Objective for division	Example
Students are able to: • solve problems involving division and adding/subtracting, using comparison bar model	**<u>Hands-on concrete stage: Demonstrate division as sharing equally into *x* groups using base ten cubes / Cuisenaire rods</u>** Example: Students use base ten cubes / Cuisenaire rod to explain the concept of division as sharing equally (x , ÷) *The sum of two numbers is 120.* *If the difference between the two numbers is 20, find the value of the two numbers.* 

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Learning Objective for division	Example
	 *Greater number = smaller number + difference* *Hence, before dividing equally into 2 numbers, the difference has to be taken off from the total to make two equal numbers.* *120 – 20 = 100* *100 ÷ 2 = 50 (smaller number)* *50 + 20 = 70 (greater number)*
	<u>Pictorial stage: Demonstrate division as sharing equally into *x* groups using rectangles and squares</u> Example: Students use base ten cubes / Cuisenaire rod to explain the concept of division as sharing equally (x , ÷) *The sum of two numbers is 120.* *If the difference between the two numbers is 20, find the value of the two numbers.*  *Greater number = smaller number + difference* *Hence, before dividing equally into 2 numbers, the difference has to be taken off from the total to make two equal numbers.*

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Learning Objective for division	Example
	120 – 20 = 100 *100 ÷ 2 = 50 (smaller number)* *50 + 20 = 70 (greater number)*
	<u>Abstract stage: division as sharing equally into *x* groups using comparison bar model</u> Example: Students use base ten cubes / Cuisenaire rod to explain the concept of division as sharing equally (x , ÷) *The sum of two numbers is 120.* *If the difference between the two numbers is 20, find the value of the two numbers.*  *120 – 20 = 100* *100 ÷ 2 = 50* *(smaller number)* *50 + 20 = 70* *(greater number)* *Greater number = smaller number + difference* *Hence, before dividing equally into 2 numbers, the difference has to be taken off from the total to make two equal bars.*

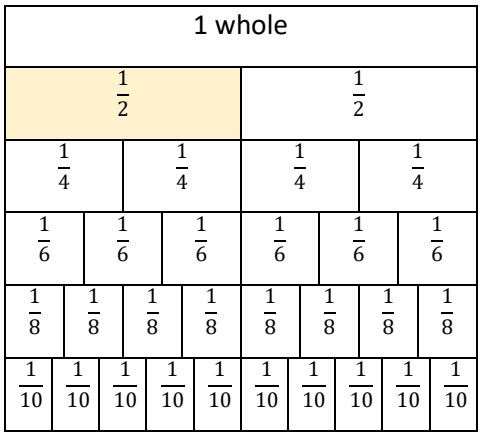
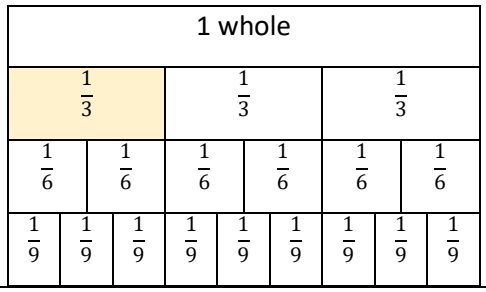
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1.2. Year 4 Fraction

Learning experience:

- recognise and show, using diagrams, families of common equivalent fractions
- count up and down in hundredths; recognise that hundredths arise when dividing an object by 100 and dividing tenths by 10
- solve problems involving increasingly harder fractions to calculate quantities, and fractions to divide quantities, including non-unit fractions where the answer is a whole number
- add and subtract fractions with the same / different denominator

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Learning Objective For fraction	Examples
Students are able to: • recognise and show, using part-part-whole bar model, families of common equivalent fractions • add and subtract equivalent fraction	**<u>Hands-on concrete stage: Cut strips of paper into equal parts to demonstrate families of common equivalent fractions</u>** Example: Students use 5 strips of paper to cut into two equal parts (halves) each. Next, cut each half into • 2 smaller equal parts to show quarters • 3 smaller equal parts to show sixths • 4 smaller equal parts to show eights • 5 smaller equal parts to show tenths Label each part.  Using the fraction strips to find equivalent of one half, $$\frac{1}{2}=\frac{2}{4} \quad , \quad \frac{1}{2}=\frac{3}{6} \quad , \quad \frac{1}{2}=\frac{4}{8} \quad , \quad \frac{1}{2}=\frac{5}{10}$$ Example: Students use 3 strips of paper to cut into three equal parts (thirds). Next, cut each third into • 2 equal parts to show sixths • 3 equal parts to show ninths. Label each part. 

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Using the fraction strips to find equivalent of one half,

$$\frac{1}{3}=\frac{2}{6} \quad , \quad \frac{1}{3}=\frac{3}{9} \quad , \quad \frac{2}{3}=\frac{4}{6} \quad , \quad \frac{2}{3}=\frac{6}{9}$$

<u>Adding of equivalent fraction:</u>

Example: $\frac{1}{2}+\frac{1}{6}=\frac{3}{6}+\frac{1}{6}=\frac{4}{6}$

Step 1: Finding pieces of paper of both fractions before joining them together to demonstrate adding.

Step 2: Check if both pieces of paper are of same size. If so, add up the number of pieces. If not, find the equivalence of the longer piece of paper. Find equivalent fraction of $\frac{1}{2}=\frac{3}{6}$. Then add.

<u>Subtraction of equivalent fraction:</u>

Example: $\frac{1}{2}-\frac{1}{6}=\frac{3}{6}-\frac{1}{6}=\frac{2}{6}$

Step 1: Find a piece of paper showing $\frac{1}{2}$.

Step 2: Since one piece of paper showing sixth ($\frac{1}{6}$) cannot be taken away from one piece of paper showing $\frac{1}{2}$, find equivalent of half into sixths. One piece of half = 3 pieces of sixth.

Step 3: Proceed to take one piece of sixth ($\frac{1}{6}$) from 3 pieces of sixth ($\frac{3}{6}=\frac{1}{2}$)

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Pictorial stage: Use fraction wall demonstrate families of common equivalent fractions

Example: Students draw equal parts on the rows of fraction wall into equal parts to demonstrate equivalent fractions

To show equivalent of half:

Step 1: Draw vertical lines to divide 2nd – 5th rows into 2 equal parts to demonstrate half.

<table>
<tr><td colspan="2">1 whole</td></tr>
<tr><td>$\frac{1}{2}$</td><td>$\frac{1}{2}$</td></tr>
<tr><td></td><td></td></tr>
<tr><td></td><td></td></tr>
<tr><td></td><td></td></tr>
<tr><td></td><td></td></tr>
</table>

Step 2: Draw vertical lines on 3rd row to divide each half into 2 equal parts to demonstrate quarters.

<table>
<tr><td colspan="4">1 whole</td></tr>
<tr><td colspan="2">$\frac{1}{2}$</td><td colspan="2">$\frac{1}{2}$</td></tr>
<tr><td>$\frac{1}{4}$</td><td>$\frac{1}{4}$</td><td>$\frac{1}{4}$</td><td>$\frac{1}{4}$</td></tr>
<tr><td colspan="2"></td><td colspan="2"></td></tr>
<tr><td colspan="2"></td><td colspan="2"></td></tr>
<tr><td colspan="2"></td><td colspan="2"></td></tr>
</table>

Step 4: Draw vertical lines on 4th row to divide each half into 3 equal parts to demonstrate sixths.

<table>
<tr><td colspan="6">1 whole</td></tr>
<tr><td colspan="3">$\frac{1}{2}$</td><td colspan="3">$\frac{1}{2}$</td></tr>
<tr><td colspan="2">$\frac{1}{4}$</td><td colspan="2">$\frac{1}{4}$</td><td colspan="2">$\frac{1}{4}$</td></tr>
<tr><td>$\frac{1}{6}$</td><td>$\frac{1}{6}$</td><td>$\frac{1}{6}$</td><td>$\frac{1}{6}$</td><td>$\frac{1}{6}$</td><td>$\frac{1}{6}$</td></tr>
<tr><td colspan="3"></td><td colspan="3"></td></tr>
<tr><td colspan="3"></td><td colspan="3"></td></tr>
</table>

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Step 5: Draw vertical lines on 5th row to divide each half into 4 equal parts to demonstrate eights.

1 whole							
$\frac{1}{2}$				$\frac{1}{2}$			
$\frac{1}{4}$		$\frac{1}{4}$		$\frac{1}{4}$		$\frac{1}{4}$	
$\frac{1}{6}$	$\frac{1}{6}$	$\frac{1}{6}$		$\frac{1}{6}$	$\frac{1}{6}$	$\frac{1}{6}$	
$\frac{1}{8}$	$\frac{1}{8}$	$\frac{1}{8}$	$\frac{1}{8}$	$\frac{1}{8}$	$\frac{1}{8}$	$\frac{1}{8}$	$\frac{1}{8}$

Step 6: Draw vertical lines on 6th row to divide each half into 5 equal parts to demonstrate tenths.

1 whole									
$\frac{1}{2}$					$\frac{1}{2}$				
$\frac{1}{4}$		$\frac{1}{4}$			$\frac{1}{4}$		$\frac{1}{4}$		
$\frac{1}{6}$	$\frac{1}{6}$	$\frac{1}{6}$			$\frac{1}{6}$	$\frac{1}{6}$	$\frac{1}{6}$		
$\frac{1}{8}$	$\frac{1}{8}$	$\frac{1}{8}$	$\frac{1}{8}$		$\frac{1}{8}$	$\frac{1}{8}$	$\frac{1}{8}$	$\frac{1}{8}$	
$\frac{1}{10}$	$\frac{1}{10}$	$\frac{1}{10}$	$\frac{1}{10}$	$\frac{1}{10}$	$\frac{1}{10}$	$\frac{1}{10}$	$\frac{1}{10}$	$\frac{1}{10}$	$\frac{1}{10}$

<u>Adding of equivalent fraction:</u>

Example: $\frac{1}{2} + \frac{1}{6} = \frac{3}{6} + \frac{1}{6} = \frac{4}{6}$

Students draw fraction wall showing halves and sixths, then shade the part showing $\frac{1}{2}$ and $\frac{1}{6}$. Since both sizes are different, find equivalent fraction of $\frac{1}{2} = \frac{3}{6}$

Count the total number of sixths.

1 whole					
$\frac{1}{2}$			$\frac{1}{2}$		
$\frac{1}{6}$	$\frac{1}{6}$	$\frac{1}{6}$	$\frac{1}{6}$	$\frac{1}{6}$	$\frac{1}{6}$

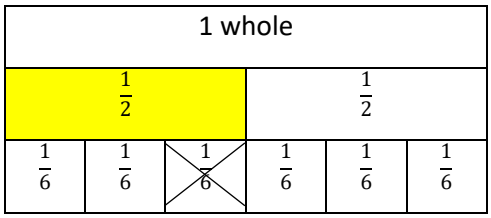
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Subtraction of equivalent fraction:

Example: $\frac{1}{2} - \frac{1}{6} = \frac{3}{6} - \frac{1}{6} = \frac{2}{6}$

Students draw fraction wall showing halves and sixths. Since both sizes are different, find equivalent fraction of $\frac{1}{2} = \frac{3}{6}$

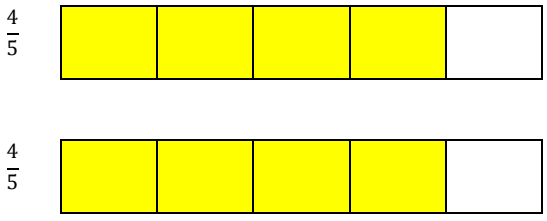
Cross out one sixth from 3 sixths



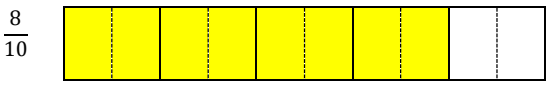
Abstract stage: Draw part-part-whole bar model to represent equivalent fractions

Example: Using part-part-whole bar model, show $\frac{4}{5} = \frac{?}{10}$

Step 1: Draw two bars showing fifths, then shade 4 parts.



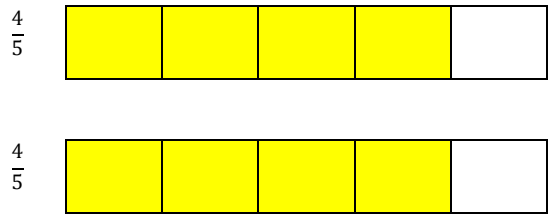
Step 2: On one of the bars, divide each part into 2 smaller equal parts to make 10 parts in total, making $\frac{4}{5} = \frac{8}{10}$



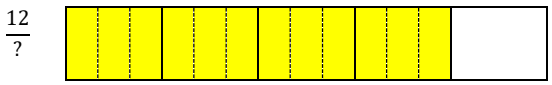
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Example: Using part-part-whole bar model, show $\frac{4}{5} = \frac{12}{?}$

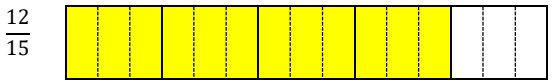
Step 1: Draw two bars showing fifths, then shade 4 parts.



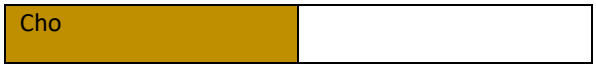
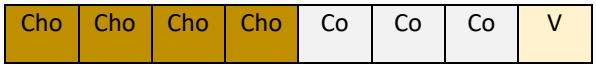
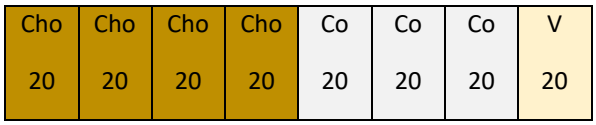
Step 2: On one of the bars, divide each *shaded* part into 3 smaller equal *shaded* parts to make 12 *shaded* parts in total



Step 3: Since it is required to make all parts equal for fraction, divide the last part into 3 smaller equal parts, making $\frac{4}{5} = \frac{12}{15}$



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Learning Objective For fraction	Examples
Students are able to: • recognise and show, using part-part-whole bar model, families of common equivalent fractions • use part-part-whole bar model to solve problems involving more challenging fractions to calculate quantities, and fractions to divide quantities, including non-unit fractions where the answer is a whole number	**<u>Hands-on concrete stage: Cut strips of paper into equal parts to solve word problem</u>** Example: Students use one strip of paper to cut into equal parts and label each part. Lynn baked some cookies. $\frac{1}{2}$ of the cookies were chocolate cookies, $\frac{3}{8}$ of the cookies were coconut cookies and the rest were vanilla cookies. If she baked 20 vanilla cookies, how many cookies did Lynn bake altogether? One strip of paper to cut: Step 1:  Step 2:  Step 3:  *After cutting the pieces, students see each part as a container for the cookies. 1 part for vanilla cookies = 1 container of 20 vanilla cookies.*

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Pictorial stage: Use fraction wall into equal parts to solve word problem

Example: Students use fraction all to divide into equal parts and label each part.

Lynn baked some cookies.
$\frac{1}{2}$ of the cookies were chocolate cookies, $\frac{3}{8}$ of the cookies were coconut cookies and the rest were vanilla cookies. If she baked 20 vanilla cookies, how many cookies did Lynn bake altogether?

Step 1:

Cho							

Step 2:

Cho	Cho	Cho	Cho	Co	Co	Co	V

Step 3:

Cho	Cho	Cho	Cho	Co	Co	Co	V
20	20	20	20	20	20	20	20

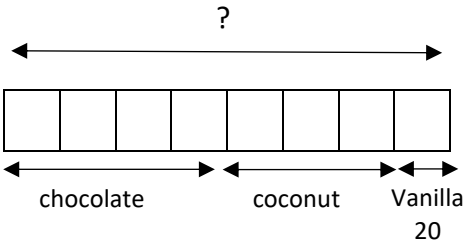
After dividing into equal pieces, students see each part as a container for the cookies. 1 part for vanilla cookies = 1 container of 20 vanilla cookies.

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Abstract stage: Draw part-part-whole bar model to represent a fraction of a number

Example: Students use part-part-whole bar model to divide into equal parts and label each part.

Lynn baked some cookies.
$\frac{1}{2}$ of the cookies were chocolate cookies, $\frac{3}{8}$ of the cookies were coconut cookies and the rest were vanilla cookies. If she baked 20 vanilla cookies, how many cookies did Lynn bake altogether?



1 unit = 20

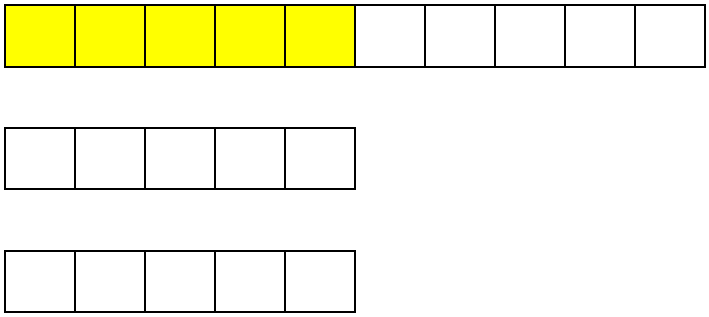
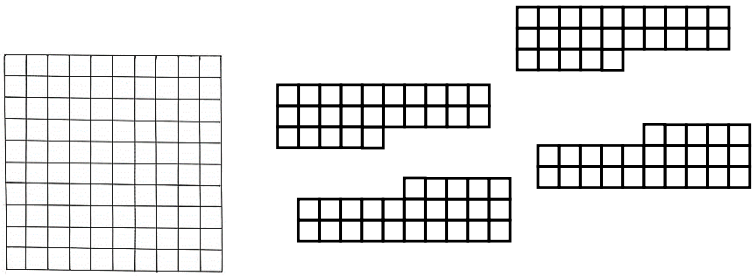
8 units = 8 x 20
*= **160***

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1.3. Year 4 Decimal tenth / hundredth

- recognise and write decimal equivalents of any number of tenths or hundreds
- recognise and write decimal equivalents to $\frac{1}{4}$, $\frac{1}{2}$, $\frac{3}{4}$
- find the effect of dividing a one- or two-digit number by 10 and 100, identifying the value of the digits in the answer as ones, tenths and hundredths
- round decimals with 1 decimal place to the nearest whole number
- compare numbers with the same number of decimal places up to 2 decimal places
- solve simple measure and money problems involving fractions and decimals to 2 decimal places

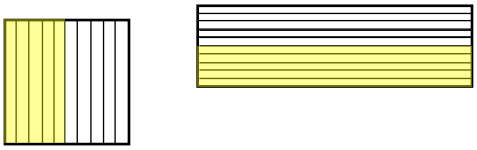
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Learning Objective for decimal	Examples
Students are able to: • recognise and write decimal equivalents to $\frac{1}{4}$, $\frac{1}{2}$, $\frac{3}{4}$ using part-part-whole bar model	**<u>Hands-on concrete stage: Cut paper strip with 10 equal division into 2 equal parts</u>** Example: Students cut strips of paper (1 whole) into 10 equal parts (10 tenths) and put them into 2 groups. $\frac{1}{2}$ of 10 tenths = 5 tenths  **<u>Hands-on concrete stage: Cut paper strip with 100 equal division into 2 / 4 equal parts</u>** $\frac{1}{2}$ of 100 hundredths = 50 hundredths $\frac{1}{4}$ of 100 hundredths = 25 hundredths 

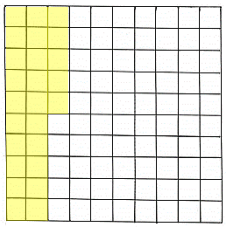
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Pictorial stage: Draw rectangles / squares to show 10 equal divisions of a whole (10 tenths) and identify half of 10 tenths

Example: Students draw rectangles / squares, divide each shape into 10 equal smaller parts, shade half



Pictorial stage: Shade on hundred grid to identify quarters of 100 equal parts of a whole



$\frac{1}{4}$ of 100 hundredths = 25 hundredths

$\frac{2}{4}$ of 100 hundredths = 50 hundredths

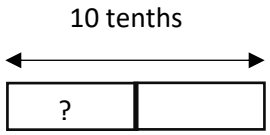
$\frac{3}{4}$ of 100 hundredths = 75 hundredths

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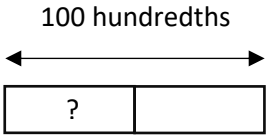
Abstract stage: Draw part-part-whole bar model

Example: Students draw part-part-whole bar model to represent tenths and hundredths

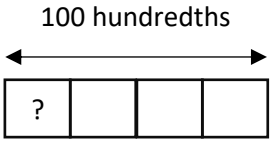
$\frac{1}{2}$ of 10 tenths = 5 tenths



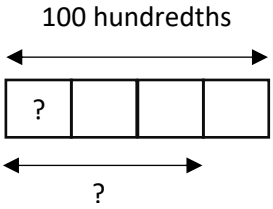
$\frac{1}{2}$ of 100 hundredths = 50 hundredths



$\frac{1}{4}$ of 100 hundredths = 25 hundredths



$\frac{3}{4}$ of 100 hundredths = 25 hundredths



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1.4. Year 4 Measurement

National Curriculum content:

Learning experience:

- convert between different units of measure [for example, kilometre to metre; hour to minute]
- measure and calculate the perimeter of a rectilinear figure (including squares) in centimetres and metres
- find the area of rectilinear shapes by counting squares
- estimate, compare and calculate different measures, including money in pounds and pence
- read, write and convert time between analogue and digital 12- and 24-hour clocks
- solve problems involving converting from hours to minutes, minutes to seconds, years to months, weeks to days

<table>
<tr><th>Learning Objective

for measurement</th><th>Examples</th></tr>
<tr><td rowspan="2">Students are able to:

• convert between different units of measure [for example, m -> cm]
• Finding fraction of measurement using part-part-whole bar model</td><td><u>Hands-on concrete stage: Using metre ruler to find fraction of a metre.</u>

Example: Students find of halves, quarters, fifths and tenths of a metre in cm

Students place different coloured tape on parts of a metre ruler to show halves, quarters, fifths and tenths

Halves – coloured tape on 50 cm and 100 cm

Quarters – coloured tape on 25cm, 50cm, 75 cm and 100cm

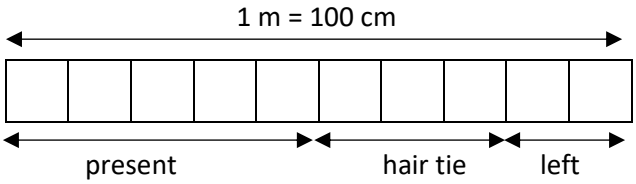
Fifths – coloured tape on 20cm, 40cm, 60cm, 80cm and 100 cm

Tenths – coloured tape on 10cm, 20cm, 30cm, 40cm, 50cm, 60cm, 70cm, 80cm, 90cm and 100 cm</td></tr>
<tr><td><u>Pictorial stage: Students draw fraction wall of a metre (100 cm)</u>

Example: Students draw equal parts on the rows of fraction wall into equal parts to demonstrate fraction of 1 m / 100 cm

<table>
<tr><td colspan="10">1 m = 100 cm</td></tr>
<tr><td colspan="5">50 cm</td><td colspan="5">50 cm</td></tr>
<tr><td colspan="2">25 cm</td><td colspan="2">25 cm</td><td colspan="2">25 cm</td><td colspan="2">25 cm</td></tr>
<tr><td colspan="2">50 cm</td><td colspan="2">50 cm</td><td colspan="2">50 cm</td><td colspan="2">50 cm</td><td colspan="2">50 cm</td></tr>
<tr><td>10 cm</td><td>10 cm</td><td>10 cm</td><td>10 cm</td><td>10 cm</td><td>10 cm</td><td>10 cm</td><td>10 cm</td><td>10 cm</td><td>10 cm</td></tr>
</table>
</td></tr>
</table>

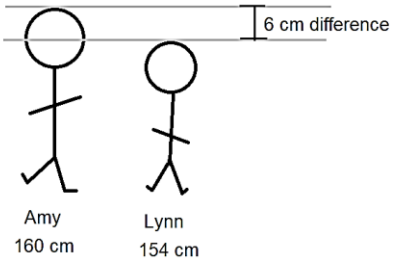
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	<u>Abstract stage: Draw part-part-whole bar model to solve measurement word problem solving</u> Example: Amy has a piece of 1 m ribbon. She cut $\frac{1}{2}$ of the ribbon to tie a present and $\frac{3}{10}$ of the ribbon for a hair tie. Find the length of the remaining piece of ribbon.  10 units = 100 cm 1 unit = 100 cm ÷ 10 = 10 cm 2 units = 2 x 10 cm = 20 cm (left)
Learning Objective **for measurement**	**Examples**
Students are able to: • convert between different units of measure [for example, kilometre to metre] • estimate, compare and calculate different measures using comparison bar model	**<u>Hands-on concrete stage: Using students and measuring tape to carry out conversion of m -> cm , cm -> m</u>** Example: Students measure their height and their peers' height in cm, then convert to m. Teacher creates some word problem using the data collected from the students, using phrases 'taller than' / 'shorter than'

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Pictorial stage: Students draw representation of their height, showing comparison with their peers

Example: Students create their comparison with peers and fill up related comparison questions



Amy is 160 cm tall.
Amy is 1.60 m tall.

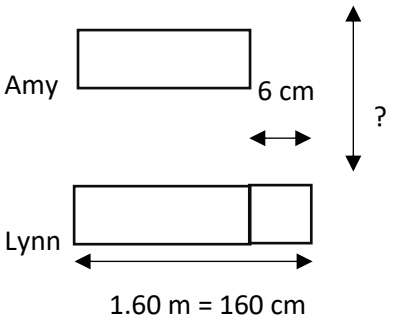
Lynn is 154 cm tall.
Lynn is 1.54 m tall.

Amy is 6 cm taller than Lynn.
Amy is 0.06 m taller than Lynn.

Lynn is 6 cm shorter than Amy.
Lynn is 0.06 m shorter than Amy.

Abstract stage: Draw comparison bar model

Example: Amy is 1.60 m tall. She is 6 cm taller than Lynn. Find their total height.



Amy's height
= 160 cm – 6 cm
= 154 cm

Total height
= 154 cm + 160 cm
= 314 cm

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Year 5

BAR MODELLING PROGRESSION

1.5. Year 5 Whole Number up to 1 000 000

1.5.1. Year 5 Number sense by counting to 1 000 000

Learning experience:

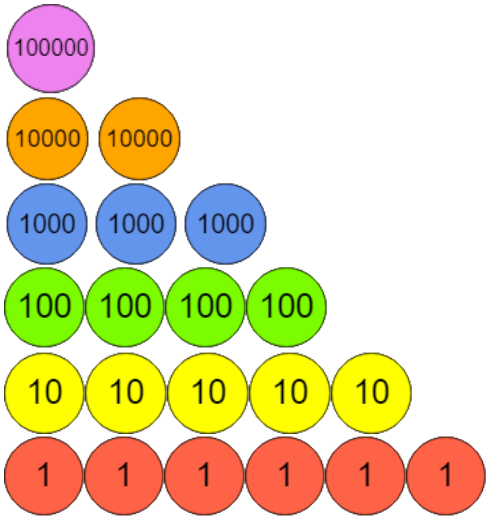
- read, write, order and compare numbers to at least 1,000,000 and determine the value of each digit
- count forwards or backwards in steps of powers of 10 for any given number up to 1,000,000
- interpret negative numbers in context, count forwards and backwards with positive and negative whole numbers, including through 0
- round any number up to 1,000,000 to the nearest 10, 100, 1,000, 10,000 and 100,000
- solve number problems and practical problems that involve all of the above
- read Roman numerals to 1,000 (M) and recognise years written in Roman numerals

<table>
<tr><th>Learning Objective
for counting numbers</th><th>Example</th></tr>
<tr><td>Students are able to:
• use place value counters to demonstrate a quantity, cardinal numbers
• compare and order numbers, using ‘<’, ‘>’ and ‘=’ signs
• count forwards or backwards in steps of powers of 10
• partitioning related to place value
• draw part-part-whole bar model to represent Cuisenaire rods and base ten blocks</td><td><u>Hands-on concrete stage: Using place value counters to carry out partitioning of a number related to place value</u>

Example: Students use different ways to partition 123 456

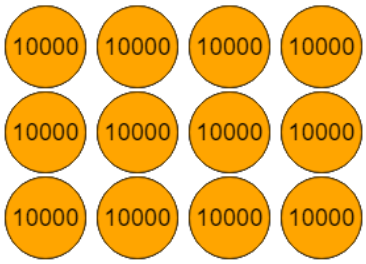
Without regrouping:

1 hundred thousand, 2 ten thousands, 3 thousands, 4 hundreds, 5 tens, 6 ones

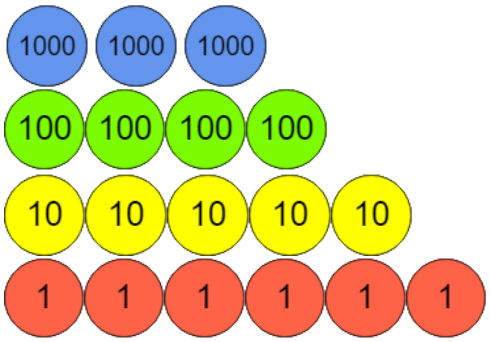
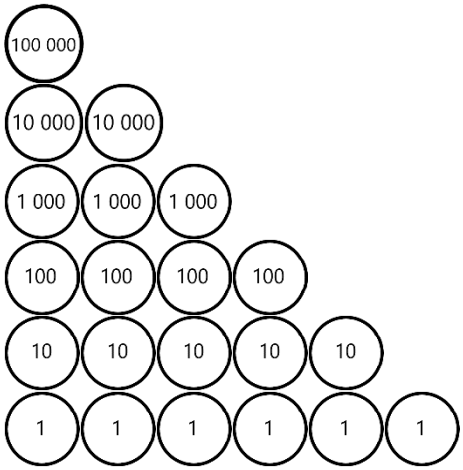
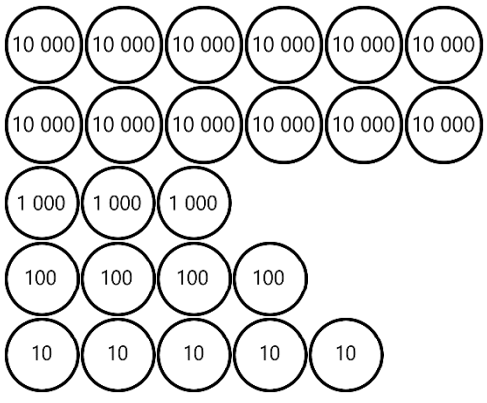


With regrouping:

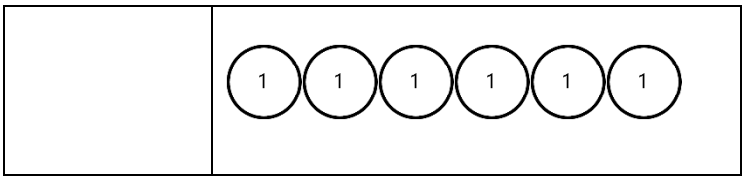
12 ten thousands, 3 thousands, 4 hundreds, 5 tens, 6 ones


</td></tr>
</table>

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Learning Objective for counting numbers	Example
	
	Pictorial stage: Using place value counters representation to carry out partitioning of a number related to place value Example: Students identify numbers from diagrams
	Without regrouping: 1 hundred thousand, 2 ten thousands, 3 thousands, 4 hundreds, 5 tens, 6 ones 
	With regrouping: 12 ten thousands, 3 thousands, 4 hundreds, 5 tens, 6 ones 

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<table>
<tr><th>Learning Objective
for counting numbers</th><th>Example</th></tr>
<tr><td rowspan="2"></td><td>
</td></tr>
<tr><td><u>Abstract stage: Using part-part-whole bar model to show different ways of number partitioning</u>

Example: Students use different ways to partition 123 456

<table>
<tr><td colspan="6">123 456</td></tr>
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<table>
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<tr><td>120 000</td><td>3 000</td><td>400</td><td>50</td><td>6</td></tr>
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1.5.2. Year 5 Addition / Subtraction of numbers with more than 4 digits

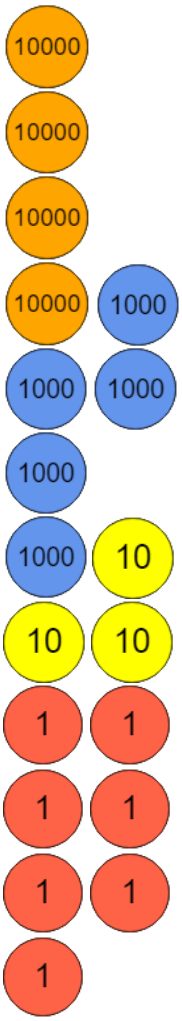
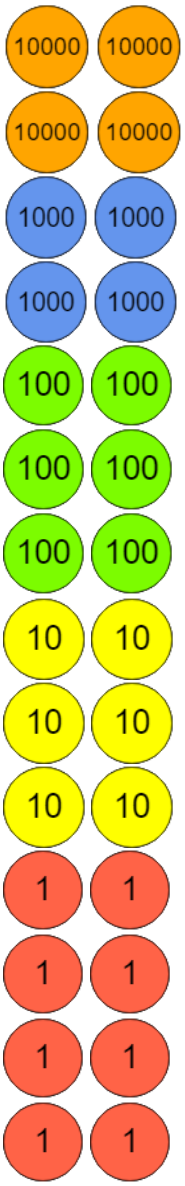
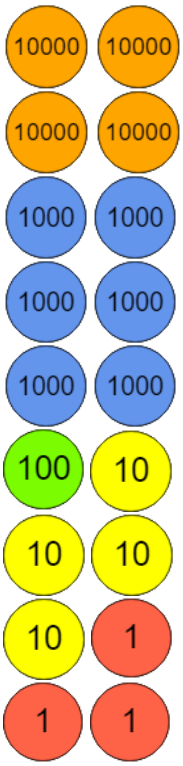
Learning experience:

- add and subtract whole numbers with more than 4 digits, including using formal written methods (columnar addition and subtraction)
- add and subtract numbers mentally with increasingly large numbers
- use rounding to check answers to calculations and determine, in the context of a problem, levels of accuracy
- solve addition and subtraction multi-step problems in contexts, deciding which operations and methods to use and why

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Learning Objective for addition / subtraction	Example
Students are able to: • Use comparison bar model to solve addition and subtraction multi-step problems in contexts, deciding which operations and methods to use and why	**<u>Hands-on concrete stage: Using place value counters to represent numbers in word problem</u>** Example: Students model the word problem question using place value counters. *Factory A produced 1 475 more robots than factory B.* *Factory B produced 369 fewer robots than factory C.* *If factory C produced 45 037 robots, how many robots did factory A produce?*

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Factory C *45 037 robots*	Factory B *369 fewer robots than factory C* *45 037 – 369 = 44 668*	Factory A *1475 more robots than factory B* *44 668 + 1475 = 46 143*
		

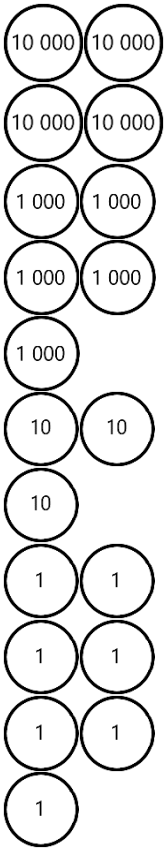
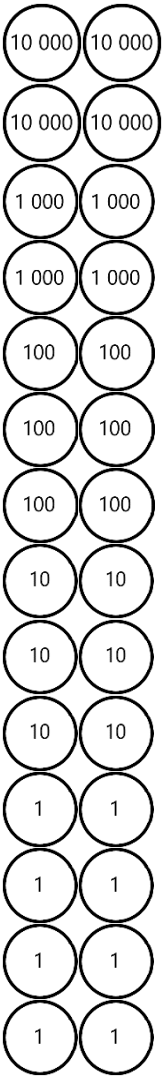
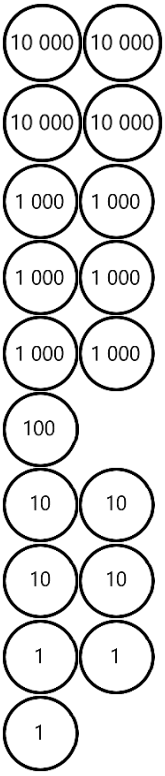
Students carry out regrouping of ones, tens and hundreds to find the number of robots produced by factory B and A.

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Pictorial stage: Drawing place value counters to represent expanded form of numbers in word problem

Example: Students model the word problem question using place value counters.

Factory A produced 1 475 more robots than factory B.
Factory B produced 369 fewer robots than factory C.
If factory C produced 45 037 robots, how many robots did factory A produce?

Factory C *45 037 robots*	Factory B *369 fewer robots than factory C* *45 037 – 369 = 44 668*	Factory A *1475 more robots than factory B* *44 668 + 1475 = 46 143*
		

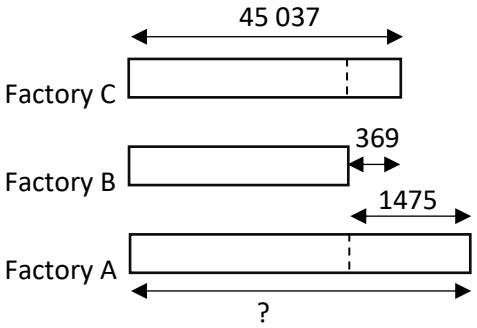
Students carry out regrouping of ones, tens and hundreds to find the number of robots produced by factory B and A.

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Abstract stage: Using comparison bar model to solve word problem

Example: Students model the word problem question using comparison bar model.

Factory A produced 1475 more robots than factory B.
Factory B produced 369 fewer robots than factory C.
If factory C produced 45 037 robots, how many robots did factory A produce?



		4	*9*	*12*	
	4	~~5~~	~~^{1}0~~	~~3~~	^{1}7
-			3	6	9
	4	4	6	6	8

		1	*1*	*1*	
	4	4	6	6	8
+		1	4	7	5
	4	6	1	4	3

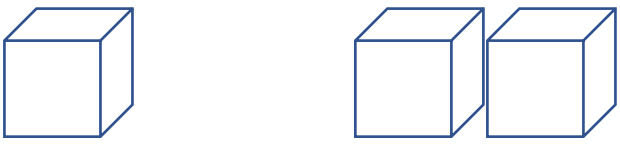
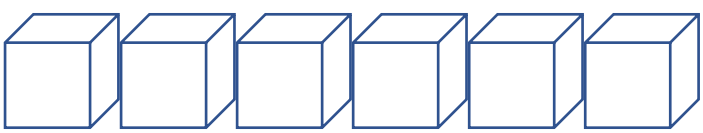
Students draw a shorter rectangle for factory B to represent a fewer number of robots, a longer rectangle for factory A than factory B to represent more robots.

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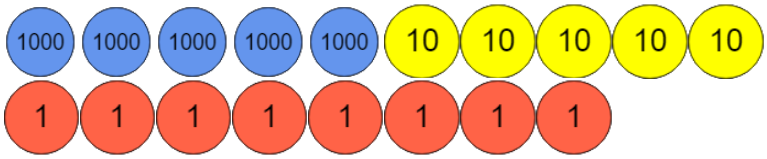
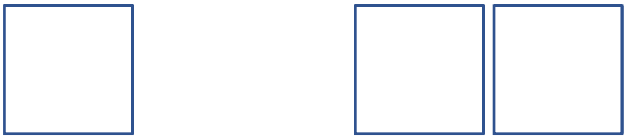
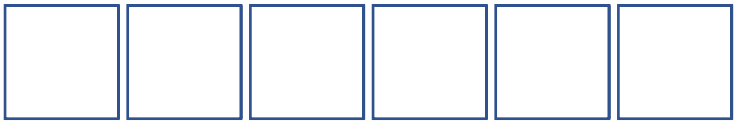
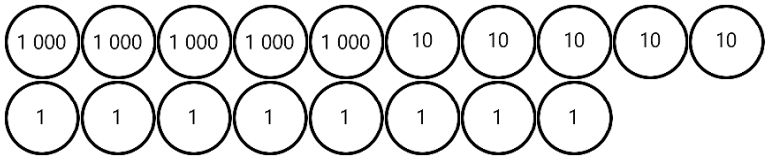
1.5.3. Year 5 Multiplication / Division of numbers

Learning experience:

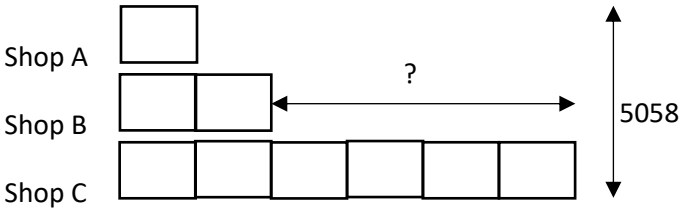
- identify multiples and factors, including finding all factor pairs of a number, and common factors of 2 numbers
- know and use the vocabulary of prime numbers, prime factors and composite (non-prime) numbers
- establish whether a number up to 100 is prime and recall prime numbers up to 19
- multiply numbers up to 4 digits by a one- or two-digit number using a formal written method, including long multiplication for two-digit numbers
- multiply and divide numbers mentally, drawing upon known facts
- divide numbers up to 4 digits by a one-digit number using the formal written method of short division and interpret remainders appropriately for the context
- multiply and divide whole numbers and those involving decimals by 10, 100 and 1,000
- recognise and use square numbers and cube numbers, and the notation for squared (^{2}) and cubed (^{3})
- solve problems involving multiplication and division, including using their knowledge of factors and multiples, squares and cubes
- solve problems involving addition, subtraction, multiplication and division and a combination of these, including understanding the meaning of the equal sign
- solve problems involving multiplication and division, including scaling by simple fractions and problems involving simple rates

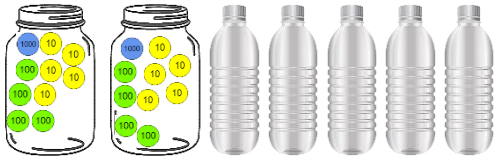
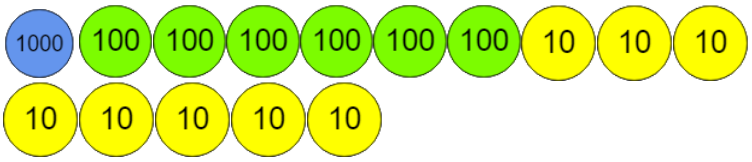
Learning Objective for multiplication / division	**Example**
Students are able to: • solve problems involving multiplication and division	**<u>Hands-on concrete stage: Using boxes and place value counters to represent numbers in word problem</u>** Example: Students model the word problem question using place value counters put in boxes. *Shop B has twice as many pencils as shop A. Shop C has 3 times as many pencils as shop B. The three shops have a total of 5058 pencils. How many more pencils are there in shop C then in shop B?* Assume same number of pencils are put in boxes.  Shop A Shop B (twice as many as Shop A)  Shop C (3 times as many pencils as Shop B)

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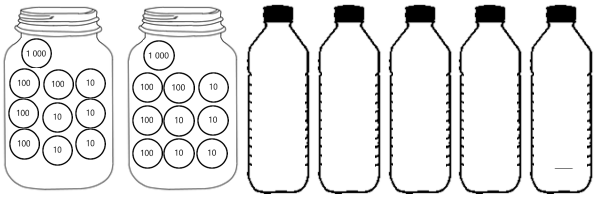
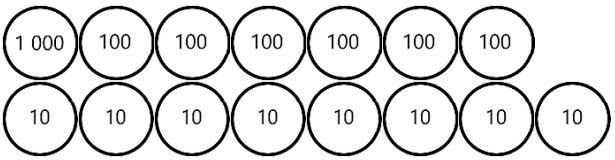
Learning Objective for multiplication / division	Example
	 9 boxes contain a total of 5058 pencils 1 box contains 5058 ÷ 9 = 562 pencils Shop C has 4 more boxes than shop B, hence 4 x 562 = 2248 pencils
	<u>Pictorial stage: Drawing place value counters to represent expanded form of numbers in word problem</u> Example: Students model the word problem question using place value counters put in boxes. *Shop B has twice as many pencils as shop A. Shop C has 3 times as many pencils as shop B. The three shops have a total of 5058 pencils. How many more pencils are there in shop C then in shop B?* Assume same number of pencils are put in boxes.  Shop A Shop B (twice as many as Shop A)  Shop C (3 times as many pencils as Shop B)  9 boxes contain a total of 5058 pencils 1 box contains 5058 ÷ 9 = 562 pencils Shop C has 4 more boxes than shop B, hence 4 x 562 = 2248 pencils
	<u>Abstract stage: Using comparison bar model to solve word problem</u>

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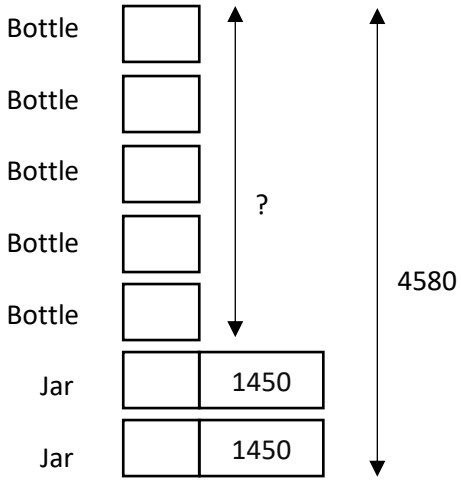
Learning Objective for multiplication / division	Example
	Example: Students draw comparison bar model to solve word problem *Shop B has twice as many pencils as shop A. Shop C has 3 times as many pencils as shop B. The three shops have a total of 5058 pencils. How many more pencils are there in shop C then in shop B?* Assume same number of pencils are put in boxes.  9 units = 5058 1 unit = 5058 ÷ 9 = 562 4 units = 4 x 562 = 2248

Learning Objective for multiplication / division	Example
Students are able to: • solve problems involving addition, subtraction, multiplication and division and a combination of these, including understanding the meaning of the equal sign	**<u>Hands-on concrete stage: Using boxes and place value counters to represent numbers in word problem</u>** Example: Students model the word problem question using place value counters put in jars and bottles. *Eve had 4580 beads. She put the beads into 2 identical jars and 5 identical bottles. There were equal number of beads in the bottles. Each jar had 1450 more beads than each bottle. How many beads were there in the 5 bottles?*   First, use counters to represent 4580 beads.

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Learning Objective for multiplication / division	Example
	Next, from the counters, put 1450 beads in each jar to show that there are 1450 more beads in each jar as compared to each bottle. Regrouping needed. 1450 + 1450 = 2900 Share the remaining beads 4580 – 2900 = 1680 into 7 containers. 1680 ÷ 7 = 240. Put 240 beads into each jar and bottle. There were 5 x 240 beads = 1200 beads in 5 bottles.
	<u>Pictorial stage: Drawing place value counters to represent expanded form of numbers in word problem</u> Example: Students model the word problem question using place value counters put in jars and bottles. *Eve had 4580 beads. She put the beads into 2 identical jars and 5 identical bottles. There were equal number of beads in the bottles. Each jar had 1450 more beads than each bottle. How many beads were there in the 5 bottles?*   First, draw 1450 counters in each jar to show that there are 1450 more beads in each jar as compared to each bottle. 1450 + 1450 = 2900 Draw remaining number 4580 – 2900 = 1680 into 7 containers. 1680 ÷ 7 = 240. Draw 240 counters into each jar and bottle. There were 5 x 240 beads = 1200 beads in 5 bottles.
	<u>Abstract stage: Using comparison bar model to solve word problem</u> Example: Students draw comparison bar model to solve word problem

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Learning Objective for multiplication / division	Example
	Eve had 4580 beads. She put the beads into 2 identical jars and 5 identical bottles. There were equal number of beads in the bottles. Each jar had 1450 more beads than each bottle. How many beads were there in the 5 bottles?  7 units + 1450 + 1450 = 4580 7 units = 4580 – 1450 – 1450 = 1680 1 unit = 1680 ÷ 7 = 240 5 units = 5 x 240 = 1200

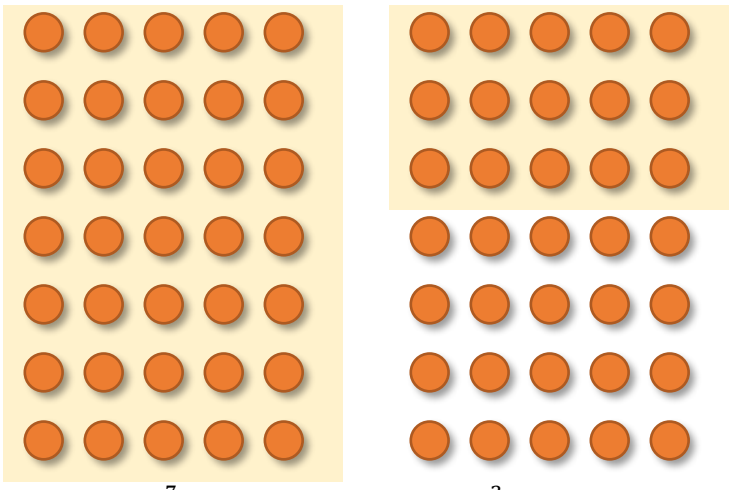
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1.6. Year 5 Fraction

Learning experience:

- compare and order fractions whose denominators are all multiples of the same number
- identify, name and write equivalent fractions of a given fraction, represented visually, including tenths and hundredths
- recognise mixed numbers and improper fractions and convert from one form to the other and write mathematical statements > 1 as a mixed number [for example, $\frac{2}{5} + \frac{4}{5} = \frac{6}{5} = 1\frac{1}{5}$]
- add and subtract fractions with the same denominator, and denominators that are multiples of the same number
- multiply proper fractions and mixed numbers by whole numbers, supported by materials and diagrams
- read and write decimal numbers as fractions [for example, $0.71 = \frac{71}{100}$]
- recognise and use thousandths and relate them to tenths, hundredths and decimal equivalents
- round decimals with 2 decimal places to the nearest whole number and to 1 decimal place
- read, write, order and compare numbers with up to 3 decimal places
- solve problems involving number up to 3 decimal places
- recognise the per cent symbol (%) and understand that per cent relates to 'number of parts per 100', and write percentages as a fraction with denominator 100, and as a decimal fraction
- solve problems which require knowing percentage and decimal equivalents of $\frac{1}{2}$, $\frac{1}{4}$, $\frac{1}{5}$, $\frac{2}{5}$, $\frac{4}{5}$ and those fractions with a denominator of a multiple of 10 or 25

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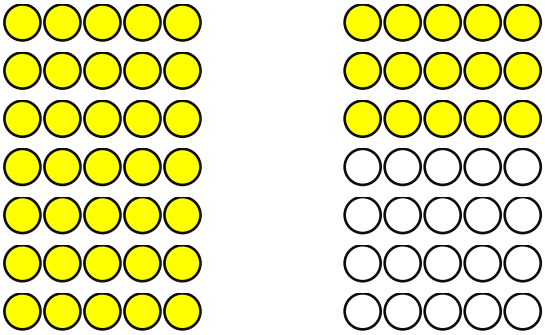
Learning Objective for fraction	Examples
Students are able to: • recognise mixed numbers and improper fractions and convert from one form to the other and write mathematical statements > 1 as a mixed number [for example, $\frac{2}{5} + \frac{4}{5} = \frac{6}{5} = 1\frac{1}{5}$] • multiply proper fractions and mixed numbers by whole numbers, supported by materials and diagrams	**Hands-on concrete stage: Using counters to find fraction of a set** Example: Students use counters to find $1\frac{3}{7}$ of 35 counters. Students change mixed number $1\frac{3}{7}$ into improper fraction $\frac{10}{7}$ Students put counters into 7 groups (denominator) to calculate the number of counters in each group. Finally, find the number of counters in 10 groups (numerator).  1 whole = $\frac{7}{7}$ of 35 = 35 $\frac{3}{7}$ of 35 = 15 $1\frac{3}{7}$ of 35 = 35 + 15 = 50

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Pictorial stage: Draw counters to find fraction of a set

Example: Students draw circles to find $1\frac{3}{7}$ of 35 circles.

Students change mixed number $1\frac{3}{7}$ into improper fraction $\frac{10}{7}$
Students put 35 circles into 7 groups (denominator) to calculate the number of counters in each group.
Finally, colour the number of counters in 10 groups (numerator).



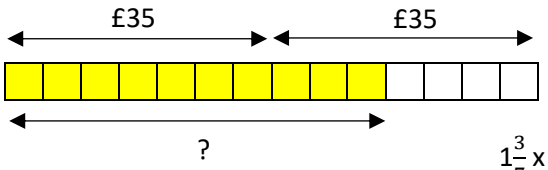
1 whole = $\frac{7}{7}$ of 35 = 35 $\quad$ $\frac{3}{7}$ of 35 = 15

$1\frac{3}{7}$ of 35 = 35 + 15 = 50

Abstract stage: Draw part-part-whole bar model to represent fraction of a set

Example: Students use part-part-whole bar model to find $1\frac{3}{7}$ of £35.

Jamie bought $1\frac{3}{7}$ m of fabric to make a sculpture for her art exhibition. The fabric cost £35 for one metre.
How much did she pay for the fabric?

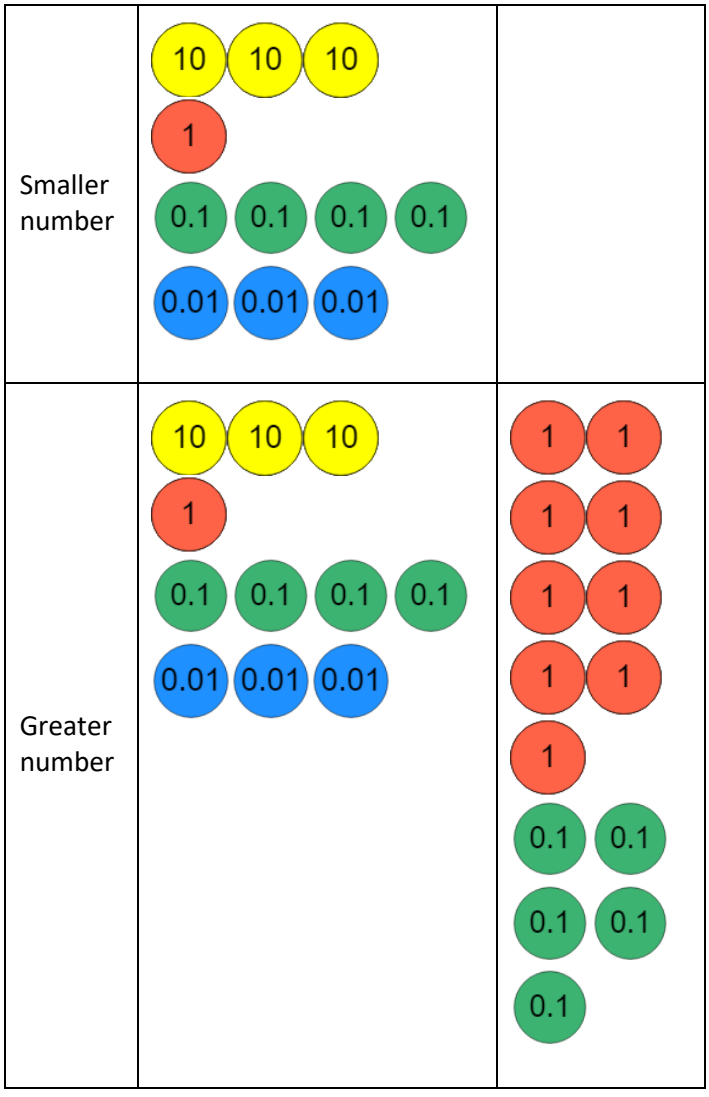


7 units = £35
1 unit = £35 ÷ 7
= £5
7 + 3 units = 10 x £5
= £50

OR

$1\frac{3}{7}$ x 35
= $\frac{10}{7}$ x 35
= $\frac{10 \text{ x } 35}{7}$
= $\frac{350}{7}$
= £50

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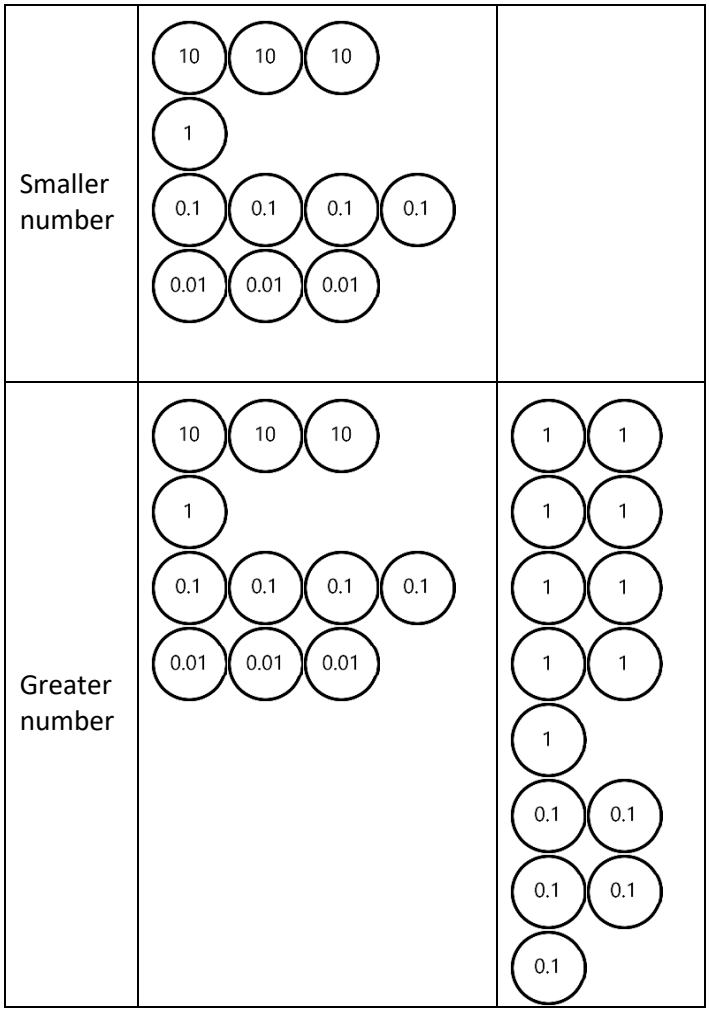
Learning Objective for decimal	Examples
Students are able to: • solve problems involving number up to 3 decimal places	**<u>Hands-on concrete stage: Using place value counters to solve problem</u>** Example: Students use place value counters to represent the decimal numbers in problem solving. *The sum of two numbers is 72.36. One of the numbers is 9.5 more than the other number. Find the smaller number.*  Step 1: Count 72.36 using counters Step 2: Place 9.5 for 'Greater Number' (*Regrouping required)* Step 3: Place the remaining counters equally for 'Smaller number' and 'Greater number'

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Pictorial stage: Draw place value counters to solve problem

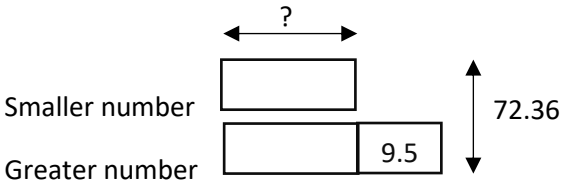
Example: Students draw place value counters to represent the decimal numbers in problem solving.

The sum of two numbers is 72.36. One of the numbers is 9.5 more than the other number. Find the smaller number.



Step 1: Draw 9.5 for 'Greater Number'
Step 2: Draw the remaining value outside of chart
(*Regrouping required)*
Step 3: Draw the remaining value equally for 'Smaller number' and 'Greater number'

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	<u>Abstract stage: Draw comparison bar model to solve problem</u> Example: Students draw comparison bar model to carry out problem solving *The sum of two numbers is 72.36. One of the numbers is 9.5 more than the other number. Find the smaller number.*  2 units + 9.5 = 72.36 2 units = 72.36 – 9.5 = 62.86 1 unit = 62.86 ÷ 2 = 31.43

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Learning Objective for percentage	Examples
Students are able to: • recognise the per cent symbol (%) and understand that per cent relates to 'number of parts per 100', and write percentages as a fraction with denominator 100, and as a decimal fraction • solve problems which require knowing percentage and decimal equivalents of $\frac{1}{2}$, $\frac{1}{4}$, $\frac{1}{5}$, $\frac{2}{5}$, $\frac{4}{5}$ and those fractions with a denominator of a multiple of 10 or 25	**Hands-on concrete stage: Cut strips of paper to find fraction/decimal/percentage equivalent** Example: Students use one strip of paper to find fraction/decimal/percentage equivalent 100% 50% | 50% 25% | 25% | 25% | 25% 20% | 20% | 20% | 20% | 20% 10% | 10% | 10% | 10% | 10% | 10% | 10% | 10% | 10% | 10% $\frac{1}{2}$ of 100% (paper strip cut into 2, take 1) = 100% ÷ 2 = 50% = 50 hundredths = 0.50 = 0.5 $\frac{1}{4}$ of 100% (paper strip cut into 4, take 1) = 100% ÷ 4 = 25% = 25 hundredths = 0.25 $\frac{3}{4}$ of 100% (paper strip cut into 4, take 3) = 3 x 25% = 75% = 75 hundredths = 0.75 $\frac{1}{5}$ of 100% (paper strip cut into 5, take 1) = 100% ÷ 5 = 20% = 20 hundredths = 0.20 = 0.2 $\frac{2}{5}$ of 100% (paper strip cut into 5, take 2) = 2 x 20% = 40% = 40 hundredths = 0.40 = 0.4 $\frac{1}{10}$ of 100% (paper strip cut into 10, take 1) = 100% ÷ 10 = 10% = 10 hundredths = 0.10 = 0.1 $\frac{1}{25}$ of 100% (paper strip cut into 25, take 1) = 100% ÷ 25 = 4% = 4 hundredths = 0.04

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Pictorial stage: Draw fraction/decimal/percentage wall to find equivalent

Example: Students draw fraction/decimal/percentage wall to find equivalent

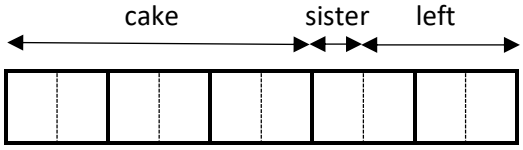
1 whole - 100%									
$\frac{1}{2}$ 50%					$\frac{1}{2}$ 50%				
$\frac{1}{4}$ 25%				$\frac{1}{4}$ 25%				$\frac{1}{4}$ 25%	
$\frac{1}{5}$ 20%			$\frac{1}{5}$ 20%			$\frac{1}{5}$ 20%			$\frac{1}{5}$ 20%
$\frac{1}{10}$ 10%	$\frac{1}{10}$ 10%	$\frac{1}{10}$ 10%	$\frac{1}{10}$ 10%	$\frac{1}{10}$ 10%	$\frac{1}{10}$ 10%	$\frac{1}{10}$ 10%	$\frac{1}{10}$ 10%	$\frac{1}{10}$ 10%	$\frac{1}{10}$ 10%

1.00 = 100%									
0.50 = 0.5 50%					0.50 = 0.5 50%				
0.25 25%				0.25 25%				0.25 25%	
0.20 = 0.2 20%			0.20 = 0.2 20%			0.20 = 0.2 20%			0.20 = 0.2 20%
0.10 = 0.1 10%	0.10 = 0.1 10%	0.10 = 0.1 10%	0.10 = 0.1 10%	0.10 = 0.1 10%	0.10 = 0.1 10%	0.10 = 0.1 10%	0.10 = 0.1 10%	0.10 = 0.1 10%	0.10 = 0.1 10%

Abstract stage: Draw part-part-whole bar model to solve problem

Example: Students draw part-part-whole bar model to carry out problem solving

Mrs Adams bought some sugar. She used $\frac{3}{5}$ of it to bake a cake and gave 25% of the remainder to her sister. What percentage of the sugar did Mrs Adams have in the end?



10 units = 100%
1 unit = 100% ÷ 10
= 10%
3 units = 3 x 10%
= 30%

OR

$\frac{3}{10} = \frac{30}{100} = 30\%$

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1.7. Year 5 Measurement

National Curriculum content:

Learning experience:

- convert between different units of metric measure [for example, kilometre and metre; centimetre and metre; centimetre and millimetre; gram and kilogram; litre and millilitre]
- understand and use approximate equivalences between metric units and common imperial units such as inches, pounds and pints
- measure and calculate the perimeter of composite rectilinear shapes in centimetres and metres
- calculate and compare the area of rectangles (including squares), including using standard units, square centimetres (cm^2) and square metres (m^2), and estimate the area of irregular shapes
- estimate volume [for example, using 1 cm^3 blocks to build cuboids (including cubes)] and capacity [for example, using water]
- solve problems involving converting between units of time
- use all four operations to solve problems involving measure [for example, length, mass, volume, money] using decimal notation, including scaling

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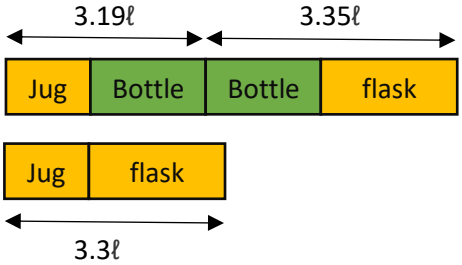
Learning Objective for measurement	Examples
Students are able to: • use all four operations to solve problems involving measure [for example, length, mass, volume, money] using decimal notation, including scaling	**<u>Hands-on concrete stage: Students use measuring cylinders to find capacity of 3 containers (less than 1ℓ)</u>** Working in groups, students label each container A, B and C. They find capacity of each container by filling each container with water to the brim, then pour the water into measuring cylinder.
	<u>Pictorial stage: Students use form word problems using information collected in concrete stage</u> Students form word problems, with diagrams using the capacity of the three containers. Helping words / phrases are to be provided for scaffolding. Example: more than / less than / difference / altogether / total / ___ times more / half / twice / thrice <u>Simple addition / subtraction word problems:</u> *Container A has a capacity of 560 mℓ of water. Container B has a capacity of 600 mℓ of water. Container C has a capacity of 710 mℓ of water. What is the total capacity of three containers?* *The total capacity of three containers is 1870 mℓ. Container A has a capacity of 560 mℓ and container B has a capacity of 600 mℓ. Find the capacity of container C.* <u>Multiplication / Division word problems</u> *The total capacity of three containers is 1870 mℓ. Container A twice as much water as container B. If container C has a capacity of 870 mℓ, what is the capacity of container A?*

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Abstract stage: Draw comparison bar model

Example: Students draw comparison bar model to carry out a given word problem.

The total capacity of a jug and a bottle is 3.19ℓ.
The total capacity of a bottle and a flask is 3.35ℓ.
The total capacity of a jug and a flask is 3.3ℓ.
Find the capacity of each container.



Jug + bottle + bottle + flask
= 3.19ℓ + 3.35ℓ
= 6.54ℓ

Given **jug + flask = 3.3ℓ**, then bottle + bottle
= 6.54ℓ - 3.3ℓ
= 3.24ℓ

Capacity of a bottle
= 3.24ℓ ÷ **2**
= 1.62ℓ

Capacity of a jug
= 3.19ℓ - 1.62ℓ
= 1.57ℓ

Capacity of a flask
= 3.35ℓ - 1.62ℓ
= 1.73ℓ

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Year 6

BAR MODELLING PROGRESSION

Bar modelling in Year 6 mathematics framework

1. Year 6 Whole Number up to 10,000,000

1.1. Year 6 Number sense by counting to 10,000,000

Learning experience:

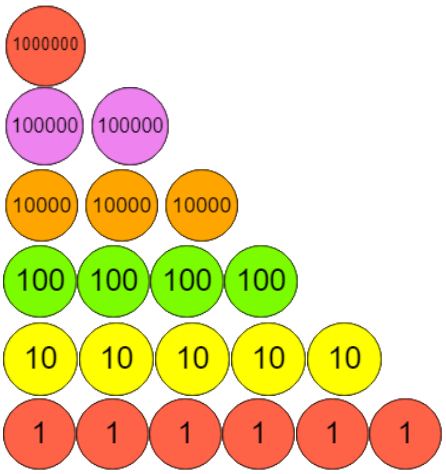
- read, write, order and compare numbers up to 10,000,000 and determine the value of each digit
- round any whole number to a required degree of accuracy
- use negative numbers in context, and calculate intervals across 0
- solve number and practical problems that involve all of the above

<table>
<tr><th>Learning Objective
for counting numbers</th><th>Example</th></tr>
<tr><td>Students are able to:
• read, write, order and compare numbers up to 10,000,000 and determine the value of each digit
• solve number and practical problems that involve all of the above</td><td><u>Hands-on concrete stage: Using place value counters to carry out partitioning of a number related to place value</u>

Example: Students use different ways to represent 1 230 456

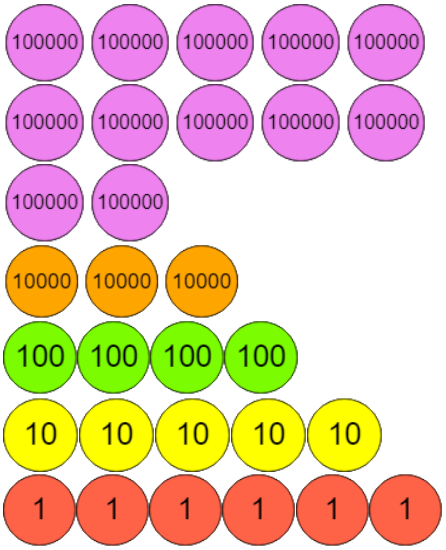
Without regrouping:

1 million,
2 hundred thousands,
3 ten thousands,
4 hundreds,
5 tens,
6 ones



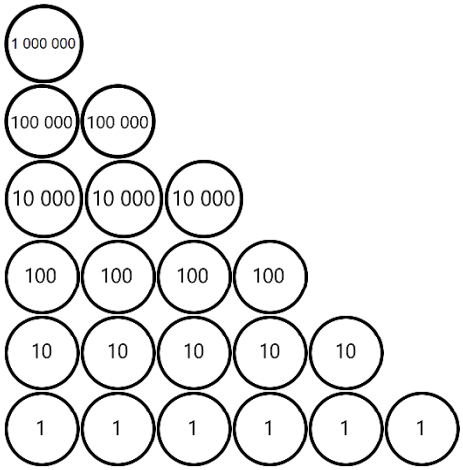
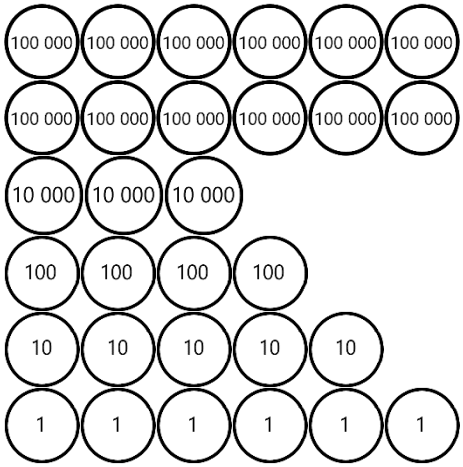
With regrouping:

12 hundred thousands,
3 ten thousands,
4 hundreds,
5 tens,
6 ones


</td></tr>
</table>

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Learning Objective for counting numbers	Example
	<u>Pictorial stage: Using place value counters representation to carry out partitioning of a number related to place value</u> Example: Students use different ways to represent 1 230 456

Without regrouping: 1 million, 2 hundred thousands, 3 ten thousands, 4 hundreds, 5 tens, 6 ones	
With regrouping: 12 hundred thousands, 3 ten thousands, 4 hundreds, 5 tens, 6 ones	

<u>Abstract stage: Using part-part-whole bar model to show different ways of number partitioning</u>

Example: Students use different ways to represent 1 230 456

1 230 456					
1 000 000	200 000	30 000	400	50	6

1 230 456				
1 200 000	30 000	400	50	6

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Learning Objective for counting numbers	Example
	<table><tr><td colspan="5">1 230 456</td></tr><tr><td>1 000 000</td><td>200 000</td><td>30 400</td><td>50</td><td>6</td></tr></table> <table><tr><td colspan="3">1 230 456</td></tr><tr><td>1 000 000</td><td>200 000</td><td>30 456</td></tr></table>

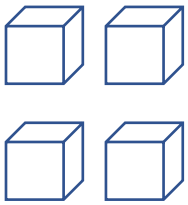
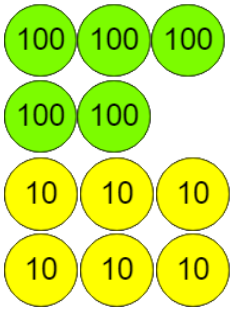
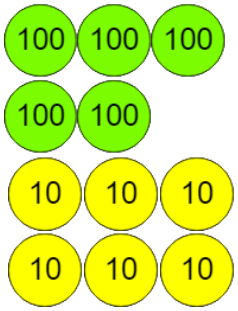
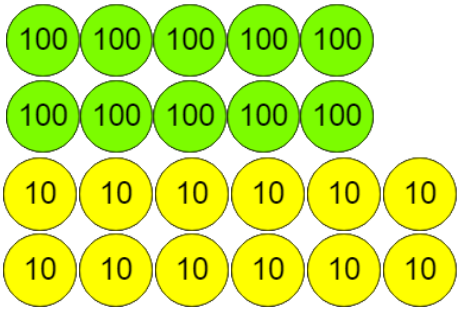
1.2. Year 6 Four operations of numbers

Learning experience:

- multiply multi-digit numbers up to 4 digits by a two-digit whole number using the formal written method of long multiplication
- divide numbers up to 4 digits by a two-digit whole number using the formal written method of long division, and interpret remainders as whole number remainders, fractions, or by rounding, as appropriate for the context
- divide numbers up to 4 digits by a two-digit number using the formal written method of short division where appropriate, interpreting remainders according to the context
- perform mental calculations, including with mixed operations and large numbers
- identify common factors, common multiples and prime numbers
- use their knowledge of the order of operations to carry out calculations involving the 4 operations
- solve addition and subtraction multi-step problems in contexts, deciding which operations and methods to use and why
- solve problems involving addition, subtraction, multiplication and division
- use estimation to check answers to calculations and determine, in the context of a problem, an appropriate degree of accuracy

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Learning Objective for addition / subtraction	Example
Students are able to: • Use comparison bar model to solve problems involving addition, subtraction, multiplication and division in contexts, deciding which operations and methods to use and why	**<u>Hands-on concrete stage: Using place value counters to represent numbers in word problem</u>** Example: Students model the word problem question using place value counters. *Jane had 4 times as much money as Timmy. How much must Jane give to Timmy so that each of them will have £560 each?* Before and after question showing total amount of money remains the same after an action of giving and receiving.

	Before	After
Jane		
Timmy		
Total amount		

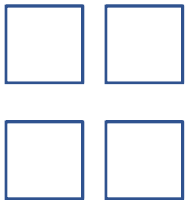
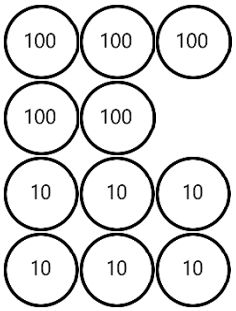
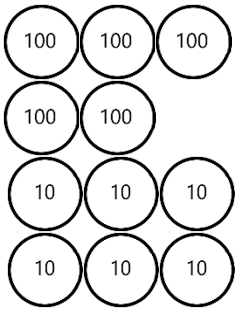
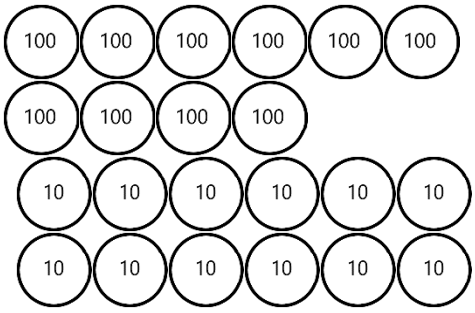
Students put all counters of Jane and Timmy together to show total. Next, they share the total counters into 5 boxes. Regrouping of tens needed.

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Pictorial stage: Drawing place value counters to represent expanded form of numbers in word problem

Jane had 4 times as much money as Timmy. How much must Jane give to Timmy so that each of them will have £560 each?

Before and after question showing total amount of money remains the same after an action of giving and receiving.

	Before	After
Jane		
Timmy		
Total amount		

Students draw all counters of Jane and Timmy together to show total. Next, they share the total counters into 5 boxes. Regrouping of tens needed.

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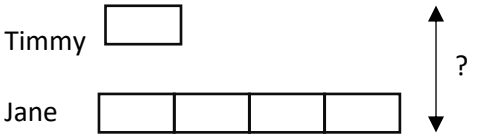
Abstract stage: Using comparison bar model to solve word problem

Example: Students model the word problem question using place value counters.

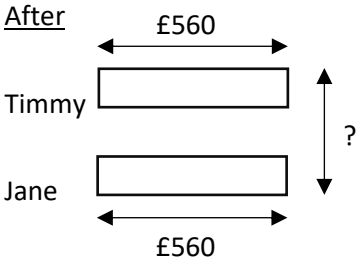
Jane had 4 times as much money as Timmy. How much must Jane give to Timmy so that each of them will have £560 each?

Before and after question showing total amount of money remains the same after an action of giving and receiving.

Before



After



Total amount of money
= £560 + £560
= £1120

5 units = £1120
1 unit = £1120 ÷ 5
= £224 (Timmy in beginning)

Amount Jane gave to Timmy *(Amount Timmy received from Jane)*
= £560 - £224
= £336

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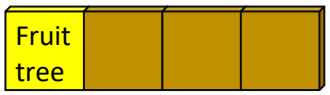
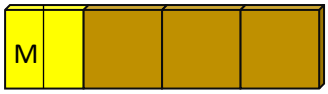
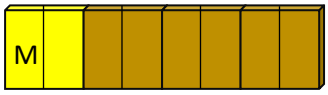
2. Year 6 Fraction

Learning experience:

- use common factors to simplify fractions; use common multiples to express fractions in the same denomination
- compare and order fractions, including fractions >1
- add and subtract fractions with different denominators and mixed numbers, using the concept of equivalent fractions
- multiply simple pairs of proper fractions, writing the answer in its simplest form [for example, $\frac{1}{4} \times \frac{1}{2} = \frac{1}{8}$]
- divide proper fractions by whole numbers [for example, $\frac{1}{3} \div 2 = \frac{1}{6}$]
- associate a fraction with division and calculate decimal fraction equivalents [for example, 0.375] for a simple fraction [for example, $\frac{3}{8}$]

- identify the value of each digit in numbers given to 3 decimal places and multiply and divide numbers by 10, 100 and 1,000 giving answers up to 3 decimal places
- multiply one-digit numbers with up to 2 decimal places by whole numbers
- use written division methods in cases where the answer has up to 2 decimal places
- solve problems which require answers to be rounded to specified degrees of accuracy
- recall and use equivalences between simple fractions, decimals and percentages, including in different contexts

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Learning Objective for fraction	Examples
Students are able to: • multiply simple pairs of proper fractions, writing the answer in its simplest form [for example, $\frac{1}{4} \times \frac{1}{2} = \frac{1}{8}$] • divide proper fractions by whole numbers [for example, $\frac{1}{3} \div 2 = \frac{1}{6}$] • associate a fraction with division and calculate decimal fraction equivalents [for example, 0.375] for a simple fraction [for example, $\frac{3}{8}$]	**<u>Hands-on concrete stage: Using square tiles to find fraction of fraction</u>** Example: Students model the word problem using square tiles. *Jack planted fruit trees on $\frac{1}{4}$ of his orchard. $\frac{1}{2}$ of fruit trees were mango trees. What fraction of the orchard was planted with mango trees?* Students use 4 square tiles to represent whole orchard as the question leads us to think about quarters (fruit trees). Out of the 4 square tiles, choose 1 colour to represent fruit trees (numerator) and 3 of same colour to represent non-fruit trees.  To find $\frac{1}{2}$ of fruit trees, students draw lines on the fruit tree tiles to represent 2 parts (denominator) and write 'M' on 1 part to represent mango trees (numerator).  To find fraction of orchard planted with mango tree, students draw lines on all tiles to make all parts equal. 

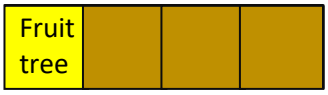
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Pictorial stage: Draw squares to find fraction of fraction

Example: Students model the word problem by drawing squares plot.

Jack planted fruit trees on $\frac{1}{4}$ of his orchard. $\frac{1}{2}$ of fruit trees were mango trees. What fraction of the orchard was planted with mango trees?

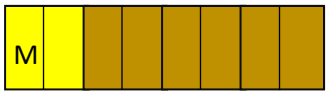
Students draw 4 square (denominator) to represent whole orchard. Out of the 4 squares, shade 1 square with 1 colour to represent fruit trees (numerator) and 3 squares of another colour to represent non-fruit trees.



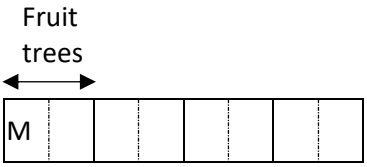
To find $\frac{1}{2}$ of fruit trees, students draw lines on the fruit tree square to represent 2 parts (denominator) and write 'M' on 1 part to represent mango trees (numerator).



To find fraction of orchard planted with mango tree, students draw lines on all tiles to make all parts equal.



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	<u>Abstract stage: Draw part-part-whole bar model to represent fraction of a fraction</u> Example: Students model the word problem by part-part-whole bar model *Jack planted fruit trees on $\frac{1}{4}$ of his orchard. $\frac{1}{2}$ of fruit trees were mango trees. What fraction of the orchard was planted with mango trees?*  $\frac{1}{2}$ of fruit trees $= \frac{1}{2} \times \frac{1}{4}$ of orchard $= \frac{1 \times 1}{2 \times 4}$ of orchard $= \frac{1}{8}$ of orchard
Learning Objective for decimal	**Examples**
Students are able to: • identify the value of each digit in numbers given to 3 decimal places and multiply and divide numbers by 10, 100 and 1,000 giving answers up to 3 decimal places • multiply one-digit numbers with up to 2 decimal places by whole numbers • use written division methods in cases where the answer has up to 2 decimal places	**<u>Hands-on concrete stage: Using weighing scale to measure concrete materials in grams and kilograms</u>** Example: Students use weighing scales and objects to measure mass. Conversion of grams to kilograms in decimal. *Suggested questions:* *The mass of a plastic container is _____ g = _____ kg.* *The mass of a toy ball is _____ g = _____ kg.* *The mass of a toy car is _____ g = _____ kg.* *Calculation:* • *The mass of a plastic container with a toy ball is _____ g = _____ kg.* • *The mass of a plastic container with a toy car is _____ g = _____ kg.* • *The mass of toy car is _____ times the mass of the toy ball.*

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Pictorial stage: Find mass of objects with pictorial aide

Example: Students study pictures of objects with mass to complete worksheet on mass.

Suggested questions:



Mass of box
560g = _____kg



Mass of toy ball
120g = _____kg



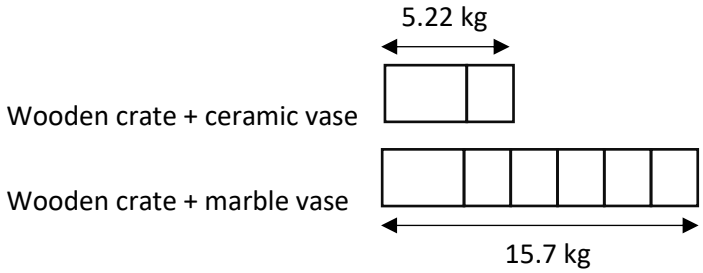
Mass of toy car
600g = _____kg

- *The mass of box with a toy ball is _____ g = _____ kg.*
- *The mass of box with a toy car is _____ g = _____ kg.*
- *The mass of toy car is _____ times the mass of the toy ball.*

Abstract stage: Draw comparison bar model to solve problem

Example: Students draw comparison bar model to carry out problem solving

The mass of a wooden crate containing a ceramic vase is 5.22 kg. The mass of the same crate containing a marble vase is 15.7 kg. The mass of the marble vase is 5 times the mass of the ceramic vase. Find the mass of the empty wooden crate.



4 units = 15.7 kg – 5.22 kg
= 10.48 kg
1 unit = 10.48 kg ÷ 4
= 2.62 kg (ceramic vase)
Mass of wooden crate
= 5.22 kg – 2.62 kg
= 2.6 kg

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3. Year 6 Ratio and Proportion

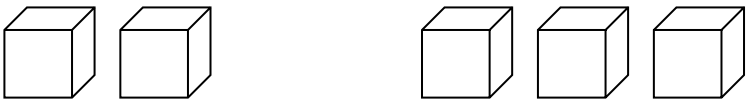
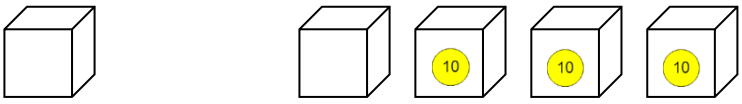
Learning experience:

- solve problems involving the relative sizes of 2 quantities where missing values can be found by using integer multiplication and division facts
- solve problems involving the calculation of percentages [for example, of measures and such as 15% of 360] and the use of percentages for comparison
- solve problems involving similar shapes where the scale factor is known or can be found
- solve problems involving unequal sharing and grouping using knowledge of fractions and multiples

<table>
<tr><th>Learning Objective
for ratio</th><th>Examples</th></tr>
<tr><td rowspan="2">Students are able to:
• solve problems involving the relative sizes of 2 quantities where missing values can be found by using integer multiplication and division facts
• solve problems involving unequal sharing and grouping using knowledge of fractions and multiples</td><td><u>Hands-on concrete stage: Using boxes and number counters to find quantities with ratio</u>

Example: Students use boxes and number counters to represent equal sets of objects in ratio

The ratio of the number of counters student A had to the number of counters Student B had was 2 : 3. After student A gave $\frac{1}{2}$ of her counters to Student B, student A had 30 counters fewer than student B. How many counters did both students have in the end?

<u>Before</u>
Student A Student B

<u>After</u>
Student A Student B


Student A had 3 boxes fewer than student B.
Student A had 30 counters fewer than student B.
3 boxes contained 30 counters.
1 box contained 30 ÷ 3 = 10 counters (Student A)
4 boxes containers 4 x 10 = 40 counters (Student B)</td></tr>
<tr><td><u>Pictorial stage: Using boxes and number counters to find quantities with equivalent ratio</u>

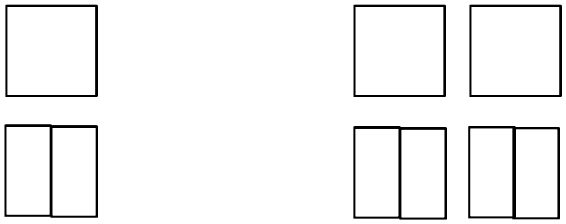
Example: Students draw squares and number counters to represent equal sets of objects in equivalent ratio</td></tr>
</table>

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The ratio of the number of counters student A had to the number of counters Student B had was 1 : 2. After student A gave $\frac{1}{2}$ of her counters to Student B, student A had 40 counters fewer than student B. How many counters did both students have in the end?

Before

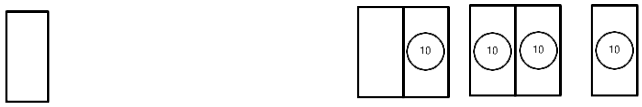
Student A | Student B



Since Student A had 1 part, and to give $\frac{1}{2}$ of her part, she had to break down the part into 2 smaller equal parts. As ratio shows the relative sizes of two quantities, Student B had to break down 2 parts into 4 smaller equal parts.

After

Student A | Student B



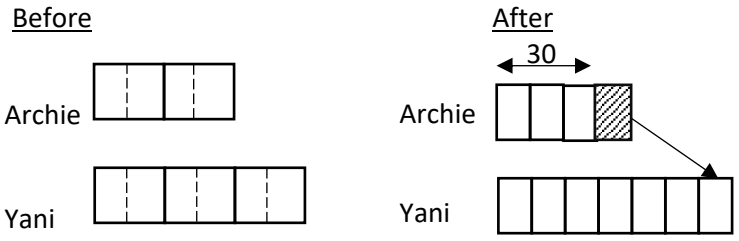
Student A had 4 boxes fewer than student B.
Student A had 40 counters fewer than student B.
4 boxes contained 40 counters.
1 box contained 40 ÷ 4 = 10 counters (Student A)
4 boxes containers 5 x 10 = 50 counters (Student B)

Abstract stage: Draw comparison bar model to solve ratio problem

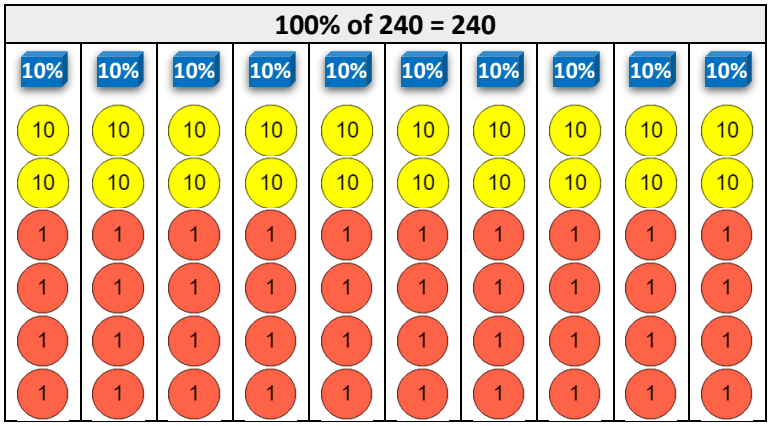
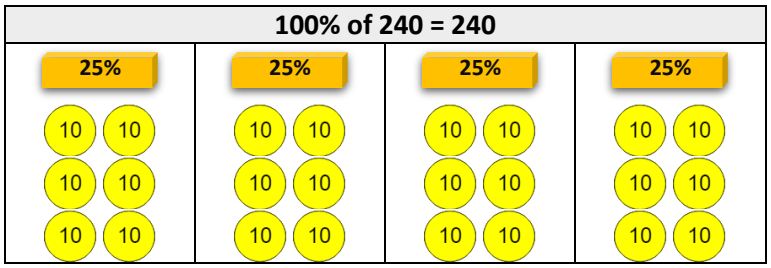
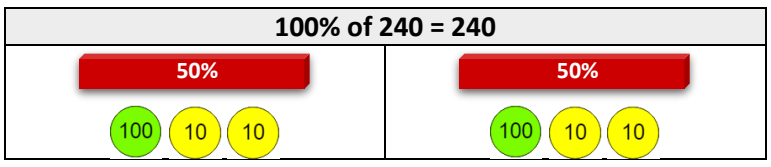
Example: Students draw comparison bar model to carry out problem solving

The ratio of the number of Archie's stickers to the number of Yani's stickers was 2 : 3. After Archie gave $\frac{1}{4}$ of his stickers to Yani, Archie had 30 stickers left. How many stickers did Yani have in the end?

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	 Before: Archie : Yani 2 : 3 4 : 6 Archie gave $\frac{1}{4}$ of his stickers After: Archie : Yani 3 : 7 3 units = 30 1 unit = 30 ÷ 3 = 10 7 units = 7 x 10 = 70 (Yani)

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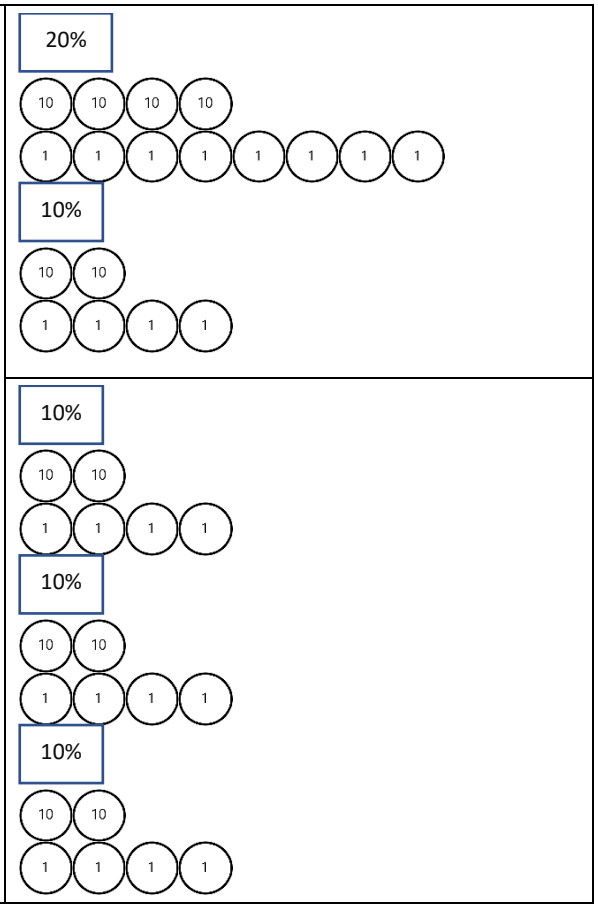
Learning Objective for percentage	Examples
Students are able to: • solve problems involving the calculation of percentages [for example, of measures and such as 15% of 360] and the use of percentages for comparison	**<u>Hands-on concrete stage: Finding % of a part and % of a whole</u>** Example: Students use fraction/percentage strips to find % of an amount Using fraction/percentage strips to represent groups, put 240 into groups.    

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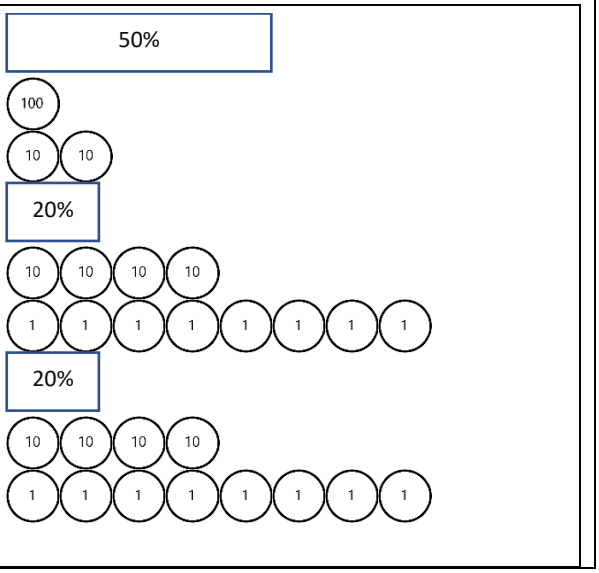
Pictorial stage: Finding % of a part and % of a whole

Example: Students use fraction/percentage parts to find % of an amount

30% of 240



90% of 240

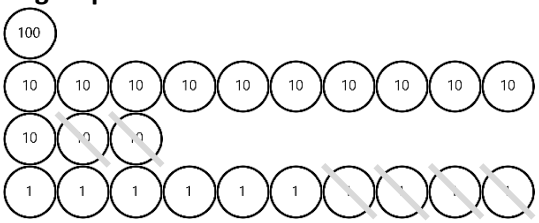


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Take 10% from 100%



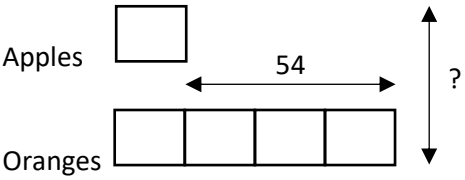
Regroup



<u>Abstract stage: Draw comparison bar model to solve problem</u>

Example: Students draw comparison bar model to carry out problem solving

A basket contains some apples and oranges. The number of apples is 25% of the number of oranges. If there are 54 more oranges than apples, how many fruits are there in the basket altogether?



25% = $\frac{1}{4}$
3 units = 54
1 unit = 54 ÷ 3
= 18 (apples)
5 units = 5 x 18
= 90

<u>OR</u>

Apples – 25%
Oranges – 100%
Difference = 100% - 25%
= 75%
75% of oranges = 54
25% of oranges
= 54 ÷ 3
= 18 (apples)
100% of oranges = 4 x 18
= 72
No. of fruits altogether
= 72 + 18
= 90

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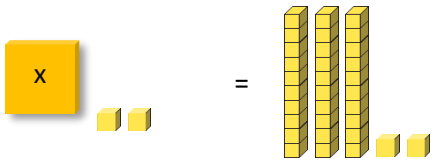
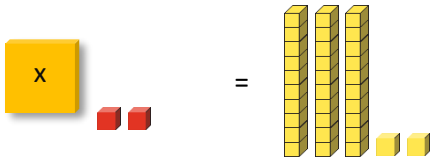
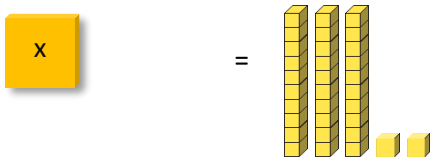
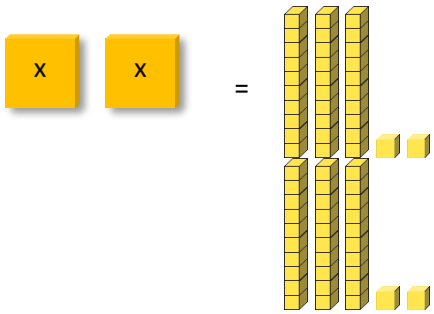
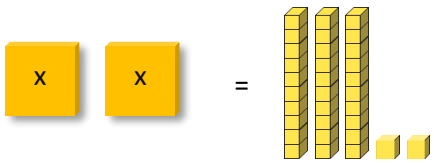
4. Year 6 Algebra

Learning experience:

- use simple formulae
- generate and describe linear number sequences
- express missing number problems algebraically
- find pairs of numbers that satisfy an equation with 2 unknowns
- enumerate possibilities of combinations of 2 variables

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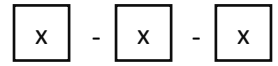
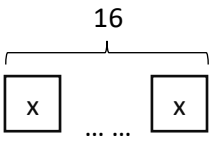
Learning Objective for measurement	Examples
Students are able to: • use simple formulae • generate and describe linear number sequences • express missing number problems algebraically • find pairs of numbers that satisfy an equation with 2 unknowns • enumerate possibilities of combinations of 2 variables	**<u>Hands-on concrete stage: Students use algebra tiles and dienes to find missing number problems.</u>** = +1 = -1

	Putting x and 2 gives 32 x + 2 = 32 x = 30
	Taking 2 away from x gives 32 x – 2 = 32 x = 34
 	Representing x = 32 2x = 2 x 32 = 64
	Putting 32 into 2 x x = 16

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Pictorial stage: Students draw representation to solve basic equation

If x = 8, find the value of the following expressions.

x + 10		8 + 10 = 18
50 – 3x		50 – 8 – 8 – 8 = 26
$\frac{x}{2} = \frac{1}{2}\ x$		8 ÷ 2 = 4
$\frac{16}{x}$		How many xs in 16? How many 8s in 16 16 ÷ 8 = 2

Abstract stage: Draw comparison bar model

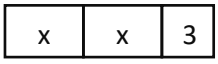
Example: Students draw comparison bar model to carry out a given word problem.

Molly is x years old now. Sally is twice as old as Molly. Peggy is 3 years older than Sally.

(a) Find the total age of the 3 girls in terms of x.
(b) If Molly is 8 years old now, find Peggy's age 2 years from now.

Molly 

Sally 

Peggy 

(a) x + x + x + x + x + 3 = 5x + 3
(b) If Molly is 8 years old now, x = 8
Peggy's age = 8 + 8 + 3
= 19
Peggy's age 2 years from now
= 19 + 2
= 21

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5. Year 6 Measurement

Learning experience:

- solve problems involving the calculation and conversion of units of measure, using decimal notation up to 3 decimal places where appropriate
- use, read, write and convert between standard units, converting measurements of length, mass, volume and time from a smaller unit of measure to a larger unit, and vice versa, using decimal notation to up to 3 decimal places
- convert between miles and kilometres
- recognise that shapes with the same areas can have different perimeters and vice versa
- recognise when it is possible to use formulae for area and volume of shapes
- calculate the area of parallelograms and triangles
- calculate, estimate and compare volume of cubes and cuboids using standard units, including cubic centimetres (cm^3) and cubic metres (m^3), and extending to other units [for example, mm^3 and km^3]

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<table>
<tr><th>Learning Objective
for measurement</th><th>Examples</th></tr>
<tr><td rowspan="2">Students are able to:

• use all four operations to solve problems involving measure [for example, length, mass, volume, money] using decimal notation, including scaling</td><td><u>Hands-on concrete stage: Students use weighing scale to find mass of each object, conversion of g / kg.</u>

Example: Students find mass of each object, in g and kg, and round off to the nearest hundred grams.</td></tr>
<tr><td><u>Pictorial stage: Students form word problems using information collected in concrete stage</u>

Example: Students create addition/subtraction/multiplication/division equations using the objects masses.

Mass of a book: 425 g ≈ 400 g (0.425 kg ≈ 0.4 kg)
Mass of a folder: 315 g ≈ 300 g (0.315 kg ≈ 0.3 kg)
Mass of an empty backpack: 1 kg 150g ≈ 1 kg 200 g (1.15 kg ≈ 1.2 kg)

The total mass of a book and a folder in the backpack is 425g + 315g + 1 kg 150 g = 1 kg 890 g

The estimated mass of book is $\frac{1}{3}$ of the backpack. The estimated mass of a backpack is 1.2 kg. Find the mass of backpack.

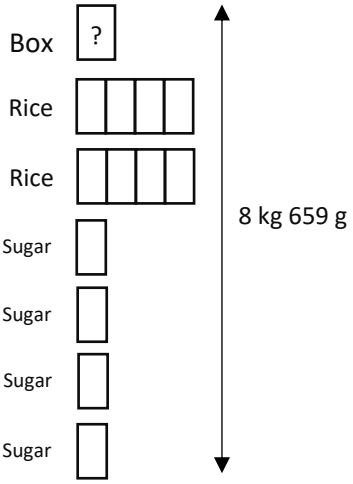
The ratio of the mass of folder to the mass of backpack is 1 : 4. The estimated mass of backpack is 1.2 kg. What is the mass of the folder?</td></tr>
</table>

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Abstract stage: Draw comparison model

Example: Students draw comparison bar model to carry out a given word problem.

The mass of a box containing 2 packets of rice and 4 packets of sugar is 8 kg 659 g. The ratio of mass of each packet of sugar to the mass of each packet of rice is 1 : 4. Given that the mass of each packet of rice is 2.68 kg, Find the mass of the empty box.

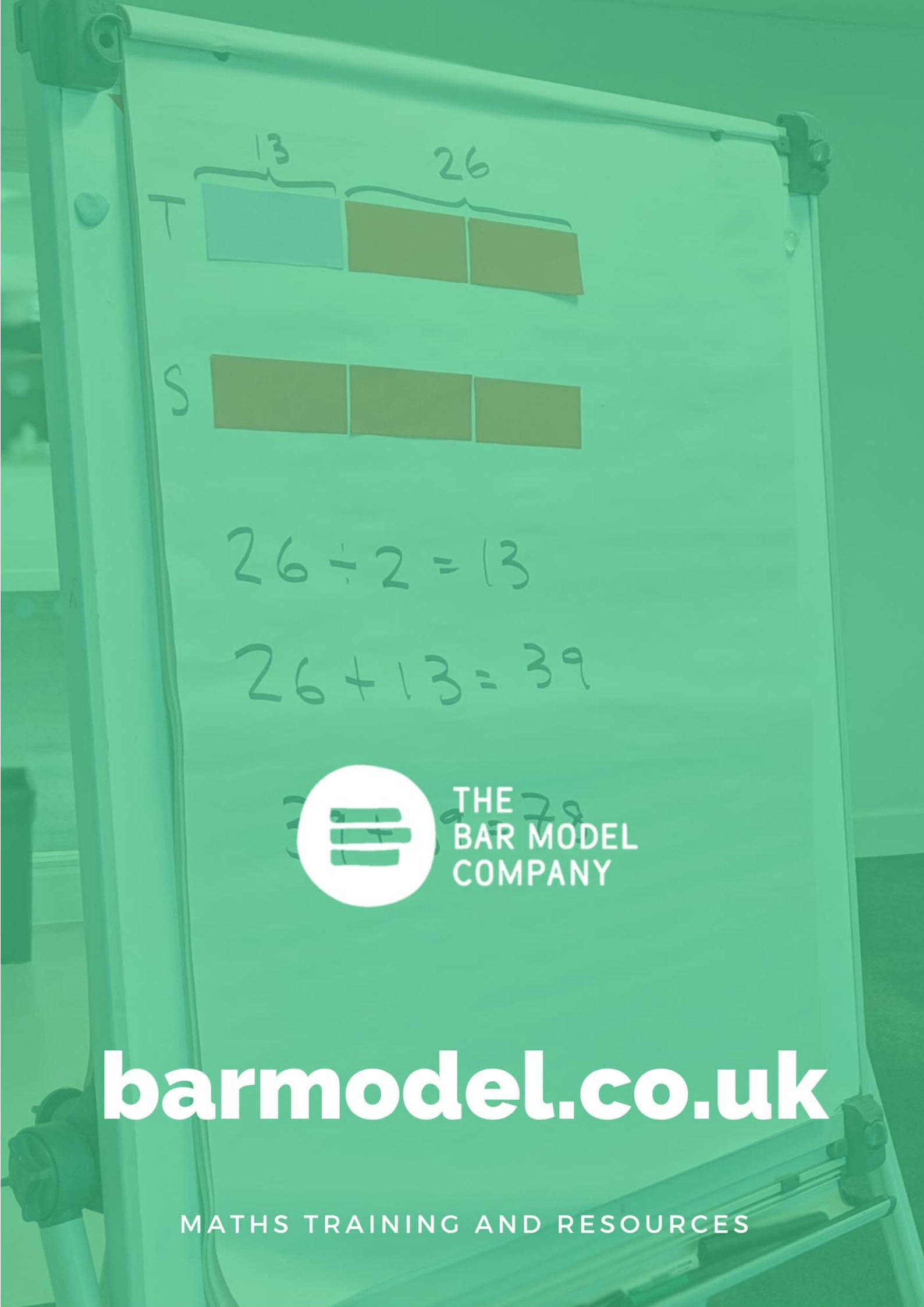


4 units = 2.68 kg

12 units = 3 x 2.68 kg
= 8.04 kg

Mass of empty box
= 8 kg 659 g – 8.04 kg
= 9.278 kg – 8.04 kg
= 0.619 kg

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